Extending Defeasibility for Propositional Standpoint Logics

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Abstract. In this paper, we introduce a new defeasible version of propositional standpoint logic by integrating Kraus et al.'s defeasible conditionals, Britz and Varzinczak's notions of defeasible necessity and distinct possibility, along with Leisegang et al.'s approach to defeasibility into the standpoint logics of Gómez Álvarez and Rudolph. The resulting logical framework allows for the expression of defeasibility on the level of implications, standpoint modal operators, and standpoint-sharpening statements. We provide a preferential semantics for this extended language and propose a tableaux calculus, which is shown to be sound and complete with respect to preferential entailment. We also establish the computational complexity of the tableaux procedure to be in PSPACE.

1 Introduction

Standpoint logics are a recently introduced family of agent-centred knowledge representation formalisms [7]. Their main feature is to allow the integration of the viewpoints of two or more agents into a single knowledge base, especially when the agents have conflicting takes on a given matter. Standpoint logics are tightly related to various systems of epistemic and doxastic logics since they build on modalities for expressing viewpoints and also assume a Kripke-style possible-worlds semantics. Sentences with standpoint-indexed modal operators such as $\Box_s\alpha$ and $\Diamond_s\alpha$ read, respectively, "from the s standpoint, it is unequivocal that α," and "from the s standpoint, it is possible that α". With standpoint-sharpening statements of the form $s \preceq t$ (which, in modal-logic terms, is an abbreviation for an axiom schema establishing the interaction between two modalities), one expresses that one standpoint is at least as specific as another, which is a way to say both standpoints agree to some extent.

In spite of allowing for the opinions upheld by agents to be in conflict without causing the knowledge base to be inconsistent, classical standpoint logics do not allow for each agent to handle exceptional cases *within* their respective standpoints. This has been partially remedied by Leisegang et al. [18], who have extended standpoint logics with both defeasible conditionals in the scope of modalities and a non-monotonic form of entailment. The resulting framework, defeasible restricted standpoint logic (DRSL), allows agents to reason about exceptions relative to their own beliefs and for defeasible consequences of a knowledge base to be derived. Nevertheless, DRSL still leaves open the question of a more general approach to defeasibility.

As pointed out by Britz and Varzinczak [3], logical languages with modalities make room for exploring defeasibility elsewhere than in conditionals: we can talk of *defeasible necessity* and *distinct possibility*, represented, respectively, by the modal operators $\mathord{\Box\!\!\!\!\sim}$ and $\mathord{\Diamond\!\!\!\!\sim}$.These enrich modal systems with defeasibility at the object level and meet a variety of applications in reasoning about defeasible knowledge, defeasible action effects, defeasible obligations, and others. It seems only natural that defeasible modalities can be fruitful in providing a formal account of the defeasible standpoints motivated above.

The goal of the present paper is to introduce Propositional Defeasible Standpoint Logic (PDSL), a new defeasible version of standpoint logic enriched with defeasibility aspects on various levels. First, we allow for a defeasible form of implication which is different from the restricted one by Leisegang et al. [18]. Second, drawing on the work of Britz and Varzinczak [3], we define defeasible versions of the standpoint modal operators found in classical standpoint logic. Finally, we extend classical standpoint logic further by allowing for the possibility of defeasible standpoint-sharpening statements.

The example below gives an idea of the level of expressivity available in PDSL.

Example 1. We consider the standpoints of vegetarians, vegans, pacifists, and environmentalists. From a vegetarian's *usual* standpoint, egg and cheese, although animal-based, are not considered unethical animal products to consume. In PDSL, this can be expressed with the sentence $\mathord{\Box\!\!\!\!\sim}_{\mathsf{Vegetarian}}((\mathsf{egg} \lor \mathsf{cheese}) \rightarrow \neg\mathsf{animal})$, which should not conflict with $\Diamond_{\mathsf{Vegetarian}}(\mathsf{egg} \land \mathsf{animal})$, an exception compatible with the vegetarian standpoint which formalises that it is possible (although unusual) to consider an egg an unethical animal product. From the vegan standpoint, though, egg and cheese are unethical animal products. This is formalised with $\Box_{\mathsf{Vegan}}((\mathsf{egg} \lor \mathsf{cheese}) \rightarrow \mathsf{animal})$, and is in line with the intuition that the vegan standpoint is a more specific version of the vegetarian one. This is captured by the sharpening sentence $\mathsf{Vegan} \leq \mathsf{Vegetarian}$. The intuition that *usually*, the vegetarian standpoint is a more specific version of the pacifist one, but allows for exceptions, e.g. those who do not eat meat only for health reasons, can be formalised as the defeasible sharpening $\mathsf{Vegetarian} \lesssim \mathsf{Pacifist}$. Among the consequences of the above, we may expect that there exists a vegan who is a typical representative of the vegan standpoint, who believes that eggs are an unethical animal product and conclude $\mathord{\Diamond\!\!\!\!\sim}_{\mathsf{Vegan}}(\mathsf{egg} \rightarrow \mathsf{animal})$. Moreover, while we may expect that typical environmentalists are vegetarians and so $\mathsf{Environmentalist} \lesssim \mathsf{Vegetarian}$ holds, we would not expect that typical environmentalists are necessarily pacifists, and so would expect that $\mathsf{Environmentalist} \lesssim \mathsf{Pacifist}$ does not hold, even though $\mathsf{Vegetarian} \lesssim \mathsf{Pacifist}$ holds.

The plan of the paper is as follows: Section 2 recalls the background and notation for the upcoming sections. Following that, and inspired by the work of Britz and Varzinczak [3] and Leisegang et al. [18], Section 3 introduces Propositional Defeasible Standpoint Logic (PDSL). In particular, we show that a preferential semantics *à la* KLM is suitable for interpreting defeasibility in PDSL and also enables us to define *preferential entailment* [17] from PDSL knowledge bases. In Section 4, we provide a tableaux-based algorithm for computing preferential entailment for PDSL, we prove its soundness and completeness, and show that its complexity is in PSPACE. Section 5 is a brief discussion on related work. Section 6 closes the paper and considers future work.

2 Preliminaries

In this section we briefly introduce the basics of classical propositional standpoint logic, as well as defeasible modalities and defeasible reasoning in modal logic, which form the basis for the logic PDSL introduced in this paper. Standpoint logic was introduced by Gómez Álvarez and Rudolph [7] for the propositional case. Given a vocabulary $\mathcal{V} = (\mathcal{P}, \mathcal{S})$, where $\mathcal{P}$ is a set of propositional atoms and $\mathcal{S}$ a finite set of standpoints including the universal standpoint $*$, the language $\mathcal{L}_{\mathbb{S}}$ over $\mathcal{V}$ is defined by:

$$\phi ::= s \leq t \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \Box_s \phi$$

where $s, t \in \mathcal{S}$ and $p \in \mathcal{P}$. Statements of the form $s \leq t$ are referred to as *standpoint sharpening statements*. The Boolean connectives $\vee$, $\rightarrow$, $\leftrightarrow$ are defined via $\neg$ and $\wedge$ in their usual manner, and for each standpoint $s \in \mathcal{S}$, we define $\Diamond_s$ as $\neg\Box_s\neg$.

A *standpoint structure* is a triple $M = (\Pi, \sigma, \gamma)$ where Π is a non-empty set of precisifications; $\sigma : \mathcal{S} \rightarrow 2^{\Pi}$ is a function such that $\sigma(*) = \Pi$ and $\sigma(s) \neq \emptyset$ for all $s \in \mathcal{S}$; $\gamma : \Pi \rightarrow 2^{\mathcal{P}}$ is a function which assigns each precisification a set of atoms. Intuitively, the mapping σ allows one to allocate to a standpoint s the set of all "reasonable ways to make s's beliefs correct", and γ assigns a set of basic propositions which are 'true' in that precisification. For a standpoint structure M and a precisification $\pi \in \Pi$, we define the satisfaction relation $\Vdash$ as follows (where $\phi, \phi_1, \phi_2 \in \mathcal{L}_{\mathbb{S}}$, $s, t \in \mathcal{S}$, and $p \in \mathcal{P}$): $M, \pi \Vdash p$ iff $p \in \gamma(\pi)$; $M, \pi \Vdash \neg\phi$ iff $M, \pi \nVdash \phi$; $M, \pi \Vdash \phi_1 \wedge \phi_2$ iff $M, \pi \Vdash \phi_1$ and $M, \pi \Vdash \phi_2$; $M, \pi \Vdash \Box_s\phi$ iff $M, \pi' \Vdash \phi$ for all $\pi' \in \sigma(s)$; $M, \pi \Vdash s \leq t$ iff $\sigma(s) \subseteq \sigma(t)$, and $M \Vdash \phi$ iff $M, \pi \Vdash \phi$ for all $\pi \in \Pi$.

Defeasible reasoning in modal logic is largely based off of similar methods in the propositional case derived from the notion of preferential consequence relations introduced by Kraus et al. [14], and rational consequence relations introduced by Lehmann and Magidor [17]. Named after the aforementioned authors, this is often called the KLM approach to defeasibility. Preferential consequence relations were considered in the modal case by Britz et al. [1, 2] and extended to include KLM-style defeasibility within modal operators themselves by Britz and Varzinczak [3]. In our paper, we build upon the defeasible multi-modal language $\mathcal{L}^{\approx}$ [3]. For a set of propositional atoms $\mathcal{P}$, the language is $\mathcal{L}^{\approx}$ defined by:

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \Box_i \phi \mid \mathrel{\approx\!\!\!\!\Box}_i \phi \mid \phi \rightsquigarrow \phi$$

where $p \in \mathcal{P}$, and $1 \leq i \leq n$, for some $n \in \mathbb{N}$. The other connectives, $\vee$, $\rightarrow$, and $\leftrightarrow$, are defined as usual. The modality $\Diamond_i$ is defined as $\neg\Box_i\neg$, and $\mathrel{\approx\!\!\!\!\Diamond}_i$ is (analogously) defined as $\neg\mathrel{\approx\!\!\!\!\Box}_i\neg$. Intuitively, $\Box_i$ indicates necessity and $\Diamond_i$ possibility (both with respect to i). Regarding the three new operators, $\mathrel{\approx\!\!\!\!\Box}_i$ is intended to indicate "usual necessity" (with respect to i), while $\mathrel{\approx\!\!\!\!\Diamond}_i$ is intended to convey "distinct" or "strong" possibility (with respect to i), and $\rightsquigarrow$ is a (possibly nested) defeasible conditional.

A preferential Kripke model is a quadruple $P = (W, R, V, \prec)$, where W is a non-empty set of worlds, $R := < R_1, \ldots, R_n >$, where each $R_i \subseteq W \times W$ is an accessibility relation on W, $V : W \longrightarrow 2^{\mathcal{P}}$ is a valuation function which maps each world to a set of propositional atoms, and $\prec$ is a strict partial order on W that is well-founded (for every $W' \subseteq W$ and every $v \in W'$, either v is $\prec$-minimal in W', or there is a

$u \in W'$ that is $\prec$-minimal in W' and $u \prec v$). Satisfaction with respect to P and a world $w \in W$ is defined as follows: For $p \in \mathcal{P}$, $P, w \Vdash p$ iff $p \in V(w)$; $P, w \Vdash \neg\phi$ iff $P, w \nVdash \phi$; $P, w \Vdash \phi_1 \wedge \phi_2$ iff $P, w \Vdash \phi$ and $P, w \Vdash \phi_2$; $P, w \Vdash \Box_i\phi$ iff $P, v \Vdash \phi$ for every $v \in W$ such that $(w, v) \in R_i$; $P, w \Vdash \tilde{\Box}_i\ \phi$ iff $P, v \Vdash \phi$ for every $v \in W$ such that $v \in \min_{\prec} R_i(w)$ (where $R_i(w) = \{w' \mid (w, w') \in R_i\}$); $P, w \Vdash \phi_1 \rightsquigarrow \phi_2$ whenever $w \notin \min_{\prec}[\![\phi_1]\!]^P$ or $w \in [\![\phi_2]\!]^P$ (where $[\![\phi_1]\!]^P$ refers to those elements v of W for which $P, v \Vdash \phi_1$, and similarly for $[\![\phi_2]\!]^P$). Finally, $P \Vdash \phi$ iff $P, w \Vdash \phi$ for every $w \in W$.

Classical multi-modal statements are interpreted in the standard way. Statements of the form $\tilde{\Box}_i\phi$ are true with respect to P and w whenever ϕ is true with respect to P and all the most typical worlds accessible from w, while statements of the form $\tilde{\Diamond}_i\phi$ are true with respect to P and w whenever ϕ is true with respect to P and at least one most typical world accessible from w. Statements of the form $\phi_1 \rightsquigarrow \phi_2$ are true in the model P when ϕ_2 is true in the most typical ϕ_1-worlds.

Britz and Varzinczak present a tableaux method for checking whether or not a statement in $\mathcal{L}^{\tilde{\Box}}$ is satisfiable in some preferential Kripke model. They prove soundness and completeness for their tableaux method, and show that it is PSPACE-complete.

3 Propositional Defeasible Standpoint Logic (PDSL)

Having dispensed with the necessary preliminaries, we now proceed to introduce Propositional Defeasible Standpoint Logic, or PDSL.

Definition 1. *Given a vocabulary $\mathcal{V} = (\mathcal{P}, \mathcal{S})$ where $\mathcal{P}$ is a set of propositional atoms and $\mathcal{S}$ is a set of standpoints, we define the set of standpoint expressions $\mathcal{E}$ over $\mathcal{S}$ as*

$$e ::= * \mid s \mid -e \mid e \cap e$$

where $s \in \mathcal{S}$. We define $\widetilde{\mathcal{L}}_{\mathbb{S}}$ over $\mathcal{V}$ (where $p \in \mathcal{P}$ and $e, d \in \mathcal{E}$) as follows:

$$\alpha ::= \top \mid p \mid e \lesssim d \mid \neg\alpha \mid \alpha \wedge \alpha \mid \Box_e\alpha \mid \tilde{\Box}_e\alpha \mid \alpha \rightsquigarrow \alpha$$

From this, we can define statements of the form $\alpha \vee \beta$, $\alpha \to \beta$ and $\alpha \leftrightarrow \beta$ in the usual way. We can also define dual symbols for both classical and defeasible standpoint modalities. That is, we define $\Diamond_e\alpha := \neg\Box_e\neg\alpha$ and $\tilde{\Diamond}_e\alpha := \neg\tilde{\Box}_e\neg\alpha$. Intuitively, $\Diamond_e\alpha$ reads "it is possible to e that α" and $\tilde{\Diamond}_e\alpha$ represents the stronger notion that "*in the most typical understandings of e's viewpoint*, it is possible that α holds". We can also define new standpoint symbols $e \cup d$ and $e \setminus d$ by as $e \cup d := -(-e \cap -d)$ and $e \setminus d := e \cap -d$, for all $e, d \in \mathcal{E}$. We are also able to define classical standpoint sharpening statements as $e \leq d := \Box_{e \setminus d}\bot$. Note that $e \leq d$ intuitively denotes that "standpoint e is a more specific version of standpoint d". That is, every precisification associated with e's standpoint can also be associated with d's standpoint. The sentence $e \lesssim d$ can then be thought of as the defeasible version of this sentence, which reads that the *most typical* precisifications associated with standpoint e are also associated with standpoint d.

The semantic structure used for defeasible standpoint modalities takes the conventions of the semantics for standpoint propositional logics [7], as well as complex standpoint expressions introduced in first-order standpoint logic [8], and adds an ordering to precisifications, where, intuitively, lower precisifications should be considered as "more typical" or more preferred states. This again follows the convention for the defeasible modalities introduced for more generalised multimodal logics [3].

Definition 2. *A **state-preferential** standpoint structure (SPSS) is a quadruple* $M = (\Pi, \sigma, \gamma, \prec)$ *where,*

- Π *is a set of precisifications.*
- $\sigma : \mathcal{E} \to 2^{\Pi}$ *is a function such that* $\sigma(*) = \Pi$*,* $\sigma(e \cap d) = \sigma(e) \cap \sigma(d)$*, and* $\sigma(-e) = \Pi \setminus \sigma(e)$*. Moreover, we require that* $\sigma(s) \neq \emptyset$ *for all* $s \in \mathcal{S}$*.*
- $\gamma : \Pi \to 2^{\mathcal{P}}$ *is a map which assigns a classical valuation to each precisification.*
- $\prec$ *is a strict partial order on* Π *such that for every subset* X *of* Π*, and every* $\pi \in X$*, either* π *is a* $\prec$*-minimal element of* X*, or there is a* $\pi' \in X$ *such that* π' *is a* $\prec$*-minimal element of* X*, and* $\pi' \prec \pi$ *(well-foundedness).*

Example 2 (Example 1 continued). Assume $\mathcal{P} = \{\mathsf{animal}, \mathsf{cheese}, \mathsf{egg}\}$ and $\mathcal{S} = \{\mathsf{Environmentalist}, \mathsf{Pacifist}, \mathsf{Vegan}, \mathsf{Vegetarian}\}$. Figure 1 depicts an example of a state preferential standpoint structure for the given vocabulary.

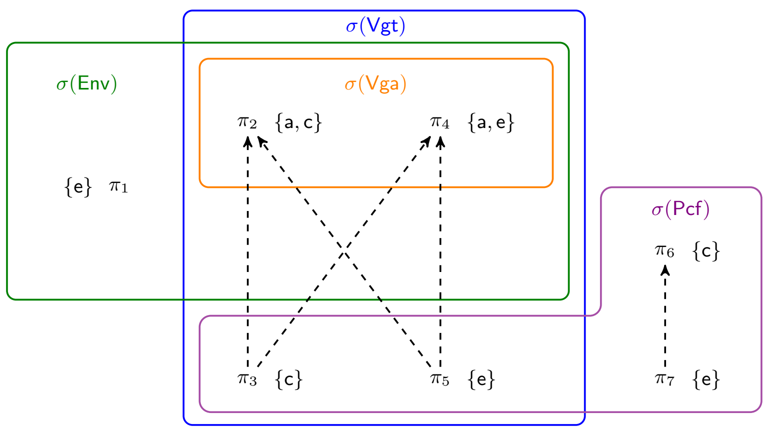

Fig. 1. A state preferential standpoint structure for $\mathcal{P} = \{\mathsf{animal}, \mathsf{cheese}, \mathsf{egg}\}$ and $\mathcal{S} = \{\mathsf{Environmentalist}, \mathsf{Pacifist}, \mathsf{Vegan}, \mathsf{Vegetarian}\}$, where $\Pi = \{\pi_i \mid 1 \leq i \leq 7\}$, $\sigma(\mathsf{Environmentalist}) = \{\pi_1, \pi_2, \pi_4\}$, $\sigma(\mathsf{Pacifist}) = \{\pi_6, \pi_7\}$, $\sigma(\mathsf{Vegan}) = \{\pi_2, \pi_4\}$, and $\sigma(\mathsf{Vegetarian}) = \{\pi_2, \pi_3, \pi_4, \pi_5\}$. Moreover, $\gamma(\pi_1) = \{\mathsf{egg}\}$, $\gamma(\pi_2) = \{\mathsf{animal}, \mathsf{cheese}\}$, $\gamma(\pi_3) = \{\mathsf{cheese}\}$, $\gamma(\pi_4) = \{\mathsf{animal}, \mathsf{egg}\}$, $\gamma(\pi_5) = \{\mathsf{egg}\}$, $\gamma(\pi_6) = \{\mathsf{cheese}\}$, and $\gamma(\pi_7) = \{\mathsf{egg}\}$. (Standpoints and atomic propositions are abbreviated for conciseness.) The strict partial order on Π is given by $\prec = \{(\pi_3, \pi_2), (\pi_3, \pi_4), (\pi_5, \pi_2), (\pi_5, \pi_4), (\pi_7, \pi_6)\}$.

We then define satisfaction for a given SPSS.

Definition 3. *For an SPSS* M *and a precisification* $\pi \in \Pi$*, we define the satisfaction relation* $\Vdash$ *inductively as follows (where* $\alpha, \alpha_1, \alpha_2 \in \mathcal{L}_{\mathbb{S}}$*,* $s, s_1, s_2 \in \mathcal{S}$ *and* $p \in \mathcal{P}$*):*

- $M, \pi \Vdash \top$.
- $M, \pi \Vdash p$ *iff* $p \in \gamma(\pi)$.
- $M, \pi \Vdash e \lesssim d$ *iff* $\min_{\prec}(\sigma(e)) \subseteq \sigma(d)$.
- $M, \pi \Vdash \neg\alpha$ *iff* $M, \pi \nVdash \alpha$.
- $M, \pi \Vdash \alpha_1 \wedge \alpha_2$ *iff* $M, \pi \Vdash \alpha_1$ *and* $M, \pi \Vdash \alpha_2$.
- $M, \pi \Vdash \Box_s \alpha$ *iff* $M, \pi' \Vdash \alpha$ *for all* $\pi' \in \sigma(s)$.
- $M, \pi \Vdash \widetilde{\Box}_s\, \alpha$ *iff* $M, \pi' \Vdash \alpha$ *for all* $\pi' \in \min_{\prec}(\sigma(s))$.
- $M, \pi \Vdash \alpha_1 \leadsto \alpha_2$ *iff* $\pi \notin \min_{\prec}[\![\alpha_1]\!]$ *or* $\pi \in [\![\alpha_2]\!]$.
- $M \Vdash \alpha$ *iff* $M, \pi \Vdash \alpha$ *for all* $\pi \in \Pi$.

We also note here several rules which the semantics introduced above satisfies in general. However, it should be noted that this list is not exhaustive by any means. Firstly, it should be clear that, since both the language and the semantics introduce notions of defeasibility on top of the existing propositional standpoint logic $\mathcal{L}_{\mathbb{S}}$, any sentences in $\mathcal{L}_{\mathbb{S}}$ which are tautologous in the original logic (as discussed by Gómez Álvarez and Rudolph [7]) are still tautologies in our case. We therefore will only discuss the defeasible parts of the logic explicitly here. We first compare the defeasible statements to their non-defeasible counterparts.

Proposition 1 (Supra-classicality). *For any SPSS M, and any $\pi \in \Pi$:*

1. $M, \pi \Vdash \Box_e \alpha \implies M, \pi \Vdash \widetilde{\Box}_e\, \alpha$.
2. $M, \pi \Vdash \widetilde{\Diamond}_e \alpha \implies M, \pi \Vdash \Diamond_e \alpha$.
3. $M, \pi \Vdash e \leq d \implies M, \pi \Vdash e \lesssim d$.
4. $M, \pi \Vdash \alpha \rightarrow \beta \implies M, \pi \Vdash \alpha \leadsto \beta$.

And in general none of the converses hold.

This tells us that $\widetilde{\Box}$, $\lesssim$ and $\leadsto$ are strictly weaker notions than their classical counterparts, while $\widetilde{\Diamond}$ is a stronger notion. We can also note how defeasible modalities affect the unions and intersections of standpoint symbols.

Proposition 2. *For an SPSS M and $\pi \in \Pi$, we have $M, \pi \Vdash \widetilde{\Box}_e\, \alpha \wedge \widetilde{\Box}_d\, \alpha \implies M, \pi \Vdash \widetilde{\Box}_{e \cup d}\, \alpha$ and $M, \pi \Vdash \widetilde{\Diamond}_{e \cup d} \alpha \implies M, \pi \Vdash \widetilde{\Diamond}_e \alpha \vee \widetilde{\Diamond}_d \alpha$. In general, the converses do not hold.*

This shows the relationship that defeasible modalities have when combining them with more "compound" standpoint symbols. It is also worth noting that this relationship is different to the case for classical standpoint modalities. For example $M \Vdash \Box_e \alpha \wedge \Box_d \alpha$ and $M \Vdash \Box_{e \cup d} \alpha$ are equivalent, while this is not the case for defeasible modalities.

Our semantics also gives us a clearer understanding of $\lesssim$. The statement $e \leq d$ can be written as a modal sharpening $\Box_{e \setminus d} \bot$, which refers to the semantic condition on a standpoint structure M where $\sigma(e) \subseteq \sigma(d)$ [8]. In the defeasible case, it is clear that an analogous translation does not occur. Consider the SPSS M in Figure 1. Clearly, $M \Vdash \mathsf{Vegetarian} \lesssim \mathsf{Pacifist}$, while $M \nVdash \Box_{\mathsf{Vegetarian} \setminus \mathsf{Pacifist}} \bot$ and $M \nVdash \widetilde{\Box}_{\mathsf{Vegetarian} \setminus \mathsf{Pacifist}}\, \bot$, since $\pi_2 \in \min_{\prec} \sigma(\mathsf{Vegetarian} \setminus \mathsf{Pacifist})$. In fact, the interpretation of the defeasible sharpening $\lesssim$ behaves as a defeasible consequence relation on the standpoint hierarchy, since $s \lesssim t$ can be interpreted semantically as stating "precisifications in s are *typically* included in t". In order to motivate this, we note that $\lesssim$ satisfies a version of the KLM rationality postulates [14].

Proposition 3. *For $e, d, g \in \mathcal{E}$, an SPSS M, and $\pi \in \Pi$ we have:*

- $M, \pi \nVdash * \lesssim e \cap -e$ *(Consistency)*
- $M, \pi \Vdash e \lesssim e$ *(Reflexivity)*
- *If $M, \pi \Vdash (e \leq d) \wedge (d \leq e)$ and $M, \pi \Vdash e \lesssim g$ then $M, \pi \Vdash d \lesssim g$* *(LLE)*
- *If $M, \pi \Vdash e \lesssim d$, and $M, \pi \Vdash d \leq g$ then $M, \pi \Vdash e \lesssim g$* *(RW)*
- *If $M, \pi \Vdash e \lesssim d$, and $M, \pi \Vdash e \lesssim g$ then $M, \pi \Vdash e \lesssim d \cap g$* *(And)*
- *If $M, \pi \Vdash e \lesssim d$, and $M, \pi \Vdash g \lesssim d$ then $M, \pi \Vdash e \cup g \lesssim d$* *(Or)*
- *If $M, \pi \Vdash e \lesssim d$, and $M, \pi \Vdash e \lesssim g$ then $M, \pi \Vdash e \cap g \lesssim d$* *(CM)*

We can also see that defeasible sharpenings satisfy an adapted version of the classical standpoint axiom (**P**) [7], given by $(e \leq d) \rightarrow (\Box_d \phi \rightarrow \Box_e \phi)$, and thus behaves as a natural extension of the classical sharpening statements.

Proposition 4. *For $e, d \in \mathcal{E}$, $\phi \in \widetilde{\mathcal{L}}_{\mathbb{S}}$, an SPSS $M = (\Pi, \sigma, \gamma, \prec)$, and any $\pi \in \Pi$, we have that $M, \pi \Vdash (e \lesssim d) \rightarrow (\Box_d \phi \rightarrow \widetilde{\Box}_e\ \phi)$.*

However, other variants of (**P**) which incorporate defeasible symbols, such as $(e \lesssim d) \rightarrow (\widetilde{\Box}_d\ \phi \rightarrow \Box_e \phi)$ or $(e \leq d) \rightarrow (\widetilde{\Box}_d\ \phi \rightarrow \widetilde{\Box}_e\ \phi)$, are not satisfied by every SPSS.

It is shown in the more general modal logic $\mathcal{L}^{\approx}$, introduced by Britz and Varzinczak [3], that the defeasible implication operator $\rightsquigarrow$ satisfies a similar set of KLM postulates. This gives us a notion of defeasible implication between Boolean formulas in the logic. However, it is worth noting that, while $\rightsquigarrow$ gives us the original KLM-style consequence for Boolean formulas in our language, this intuition does not follow when we combine it with standpoint modalities in our language. In particular, when we bound a defeasible implication $p \rightsquigarrow q$ with some defeasible or non-defeasible standpoint modality, we may expect this to tell us something about a standpoints defeasible beliefs. For example, we may expect $\Box_s(p \rightsquigarrow q)$ to tell us that "*from s's standpoint, the most typical instances of p are instances of q*". The following example shows us that this is not the case.

Example 3. Consider an SPSS $M = (\Pi, \sigma, \gamma, \prec)$ over propositional atoms p and q and a single standpoint s defined as follows: $\Pi = \{\pi_1, \pi_2\}$, $\sigma(s) = \{\pi_2\}$, $\gamma(\pi_1) = \{p, q\}$, $\gamma(\pi_2) = \{p\}$, and $\pi_1 \prec \pi_2 \prec \pi_3$. Then note $M \Vdash \Box_s(p \rightsquigarrow q)$ iff $M, \pi \Vdash p \rightsquigarrow q$ for all $\pi \in \sigma(s)$. That is, $\pi \in \sigma(s)$ implies $\pi \notin \min_{\prec} [\![p]\!]$ or $\pi \in [\![q]\!]$. Since in the above model $\sigma(s) \cap \min_{\prec} [\![p]\!] = \emptyset$, we have $M \Vdash \Box_s(p \rightsquigarrow q)$, even though the only precisification in $\sigma(s)$ (and therefore the minimal one) violates $p \rightarrow q$, and so s's standpoint intuitively does not believe that the most typical instances of p are instances of q.

Thus, defeasible implication $\widetilde{\mathcal{L}}_{\mathbb{S}}$ does not serve to represent standpoints holding defeasible beliefs, and a separate semantics is introduced for this problem in [18]. We also note that $\rightsquigarrow$ does not provide an intuitive account of defeasible consequences for modal statements. For example, $M \Vdash \Box_e \alpha \rightsquigarrow \beta$ if and only every $\pi \in \min_{\prec} [\![\Box_e \alpha]\!] \subseteq [\![\beta]\!]$. However, it is clear that by definition, either $[\![\Box_e \alpha]\!] = \emptyset$ or $[\![\Box_e \alpha]\!] = \Pi$. In the first case, $M \Vdash \Box_e \alpha \rightsquigarrow \beta$ trivially; in the second, $M \Vdash \Box_e \alpha \rightsquigarrow \beta$ iff $M, \pi \Vdash \beta$ for every $\pi \in \min_{\prec} \Pi$, which is equivalent to $M \Vdash \widetilde{\Box}_*\ \beta$. We therefore treat $\rightsquigarrow$ as a part of the language which is useful for describing defeasible implication between Boolean statements, but not as an intuitively meaningful statement outside of these bounds.

4 Satisfiability Checking and Preferential Entailment

In this section we address the notion of preferential satisfiability and preferential entailment in the semantics for $\widetilde{\mathcal{L}}_{\mathbb{S}}$. We differentiate between local and global satisfaction, as is defined below.

Definition 4. *Let α be a sentence in $\widetilde{\mathcal{L}}_{\mathbb{S}}$. We say that α is **locally satisfiable** if there exists an SPSS $M = (\Pi, \sigma, \gamma, \prec)$ for which there is some precisification $\pi \in \Pi$ such that $M, \pi \Vdash \alpha$. We say that α is **globally satisfiable** if there exists an SPSS M such that $M \Vdash \alpha$. For any finite set $A \subseteq \widetilde{\mathcal{L}}_{\mathbb{S}}$, we say that A is locally (resp. globally) satisfiable if $\bigwedge A$ is locally (resp. globally) satisfiable.*

Closely related to this is the notion of preferential entailment, which extends the notion of preferential entailment found in the propositional case by Kraus et al. [14], and in the case for defeasible modalities in logic **K** by Britz et al. [1].

Definition 5. *Consider a finite knowledge base $\mathcal{K} \subseteq \widetilde{\mathcal{L}}_{\mathbb{S}}$, and a sentence $\alpha \in \widetilde{\mathcal{L}}_{\mathbb{S}}$. We say that $\mathcal{K}$ **preferentially entails** α or write $\mathcal{K} \vDash_P \alpha$ if, for every SPSS M such that $M \Vdash \phi$ for all $\phi \in \mathcal{K}$, we have $M \Vdash \alpha$.*

It is noted by Britz et al. [2] that preferential entailment defined this way induces a monotonic consequence operator. Therefore while the defeasible symbols we introduce are non-monotonic on the object-level, the entailment-level reasoning remains monotonic. The following propositions show that both global satisfiability and preferential entailment can be expressed in terms of local satisfiability.

Proposition 5. *Consider a globally satisfiable knowledge base $\mathcal{K} \subseteq \widetilde{\mathcal{L}}_{\mathbb{S}}$ and a sentence $\alpha \in \widetilde{\mathcal{L}}_{\mathbb{S}}$. Then α is globally satisfiable iff $\Box_* \alpha$ is locally satisfiable. Furthermore, $\mathcal{K} \vDash_P \alpha$ iff $\Box_*(\bigwedge \mathcal{K}) \land \neg\alpha$ is not locally satisfiable.*

Our first method for checking preferential satisfiability is via translating our logic into classical propositional standpoint logic as described by Gómez Álvarez and Rudolph [7], including complex standpoint expressions.

Definition 6. *Let $M = (\Pi, \sigma, \gamma, \prec)$ be a state preferential standpoint structure over the vocabulary $\mathcal{V} = (\mathcal{P}, \mathcal{S})$. Then the translation $T(M) = (\Pi, \sigma', \gamma)$ of M is a standpoint structure over the vocabulary $\mathcal{V} = (\mathcal{P}, \mathcal{S} \cup \tilde{\mathcal{E}})$ where: Π and γ are the same in each structure, $\tilde{\mathcal{E}} = \{\tilde{e} \mid e \in \mathcal{E}\}$, and σ' is defined by $\sigma'(s) = \sigma(s)$ for $s \in \mathcal{S}$ and $\sigma'(\tilde{e}) = \min_{\prec} \sigma(e)$ for $\tilde{e} \in \tilde{\mathcal{E}}$. The value of σ' is extended to complex standpoint expressions inductively, as is done in the literature [8].*

We can then express the satisfiability of defeasible modalities and sharpenings in terms of classical standpoint logic.

Proposition 6. *For any SPSS M, and any sentence α in classical standpoint logic, we have that $M \Vdash \widetilde{\Box}_e\, \alpha$ iff $T(M) \Vdash \Box_{\tilde{e}}\alpha$, and $M \Vdash e \lesssim d$ iff $T(M) \Vdash \tilde{e} \leq d$.*

This can then give us a means to determine satisfiability, using existing methods in standpoint logics [7, 8].

Corollary 1. *For any sentence $\alpha \in \widetilde{\mathcal{L}}_{\mathbb{S}}$ not containing the symbol "$\rightsquigarrow$", we have that α is locally satisfiable iff $T(\alpha)$ is satisfiable in classical standpoint semantics, where $T(\alpha)$ is the sentence formed by replacing every instance of $\widetilde{\Box}_e$ occurring in α with $\Box_{\tilde{e}}$ and every subsentence of the form $e \lesssim d$ with $\tilde{e} \leq d$.*

This approach however, has two weaknesses. Firstly, it does not provide a method for determining the satisfiability of sentences in $\widetilde{\mathcal{L}}_{\mathbb{S}}$ containing "$\rightsquigarrow$". Secondly, the set $\tilde{\mathcal{E}}$ as given in Definition 6 is infinite. Since $\mathcal{E}$ behaves as a Boolean algebra of subsets, we could clearly reduce the size of $\tilde{\mathcal{E}}$ to account for equivalences between Boolean formulas. However, for any complex standpoint expression e, we cannot in general express $\tilde{e}$ in terms of other, smaller standpoint symbols in $\tilde{\mathcal{E}}$. For example, we cannot express $\widetilde{e \cap d}$ in terms of $\tilde{e}$ and $\tilde{d}$. This means that if $|\mathcal{S}| = n$, then the size of $\tilde{\mathcal{E}}$ (once reduced to account for Boolean equivalences) may still be as big as 2^{2^n}, since we need to consider a new standpoint for every non-equivalent Boolean combination of the original standpoints. This potential double exponential blow-up means that the given translation is not an effective means for determining satisfiability in $\widetilde{\mathcal{L}}_{\mathbb{S}}$. However, in the restricted case where we only allow sentences with atomic standpoint indexes (i.e., where we restrict $\mathcal{E}$ to the set $\mathcal{S} \cup \{*\}$), the size of $\tilde{\mathcal{E}}$ is only $2n+2$. Therefore, since satisfiability for propositional standpoint logic is NP-complete [7], the translation above provides an NP-complete means for checking satisfiability in the setting where only atomic standpoint indexes occur in α.

However, in the general case for determining satisfiability and preferential entailments, we need to turn to other methods in order to avoid a double exponential blow-up in standpoint symbols. We therefore propose a tableau algorithm for computing whether a given statement in $\widetilde{\mathcal{L}}_{\mathbb{S}}$ is locally satisfiable. Our tableau is semantic in nature, and follows closely conventions for semantic tableau in related modal logics [3, 4, 12]. To this end, we introduce a normal form for sentences in $\widetilde{\mathcal{L}}_{\mathbb{S}}$.

Definition 7. *If a standpoint expression $c \in \mathcal{E}$ is of the form $c = s_1 \cap s_2 \cap ... \cap s_k \cap -s_{k+1} \cap ... \cap -s_l$ where $s_1, ..., s_l \in \mathcal{S}$ we call it a **standpoint conjunct**. If $k = 0$, for the sake of the tableau we add $*$ to the conjunct so that $c = * \cap -(s_{k+1} \cup ... \cup s_m)$.*

*For any $e \in \mathcal{E}$, we say that e is in **disjunctive normal form** (DNF) if it is of the form $e = c_1 \cup ... \cup c_m$ where $c_1, ..., c_m$ are all standpoint conjuncts. We then say a formula $\alpha \in \widetilde{\mathcal{L}}_{\mathbb{S}}$ is in **index normal form** (INF) if every standpoint expression which appears in ϕ is in disjunctive normal form.*

Since the logic of standpoint expressions operates as a Boolean algebra of subsets, the well-known result that each standpoint expression has an equivalent expression in DNF holds. Therefore, when we check for satisfiability using the following tableau method, we first assume each formula is in INF. In general, in the tableau, an indexed lowercase letter c will denote a standpoint-literal conjunction, while an indexed letter s will denote an atomic standpoint. e and d refer to any standpoint in DNF. This allows us to describe our tableau system.

Definition 8 ([3]). *If $n \in \mathbb{N}$ and $\alpha \in \widetilde{\mathcal{L}}_{\mathbb{S}}$, then $n :: \alpha$ is a **labelled sentence**.*

Intuitively, the labelled sentence $n :: \alpha$ indicates semantically that there is a precisification n in the model such that α holds at n.

Definition 9. *A **skeleton** is a function $\Sigma : \mathcal{E} \to 2^{\mathbb{N}}$. A **preference** relation $\prec$ is a binary relation on $\mathbb{N}$.*

A skeleton intuitively assigns each standpoint a set of precisifications, while the preference relation in the tableau acts to construct the preference ordering in an SPSS. Both Σ and $\prec$ are built cumulatively, and so at the beginning of the tableau we assume $\prec = \emptyset$ and $\Sigma(e) = \emptyset$ for all $e \in \mathcal{E}$.

Definition 10 ([3]). *A branch is a tuple $(\mathcal{B}, \Sigma, \prec)$ where $\mathcal{B}$ is a set of labelled sentences, Σ is a skeleton and $\prec$ is a preference relation.*

Definition 11 ([3]). *A **tableau rule** is of the form*

$$(\rho)\frac{\mathcal{N} : \Gamma}{\mathcal{D}_1; \Gamma_1|...|\mathcal{D}_k; \Gamma_k}$$

*where $\mathcal{N} : \Gamma$ is the **numerator** and $\mathcal{D}_\infty; \Gamma_1|...|\mathcal{D}_\infty; \Gamma_1$ is the **denominator**.*

As in [3], $\mathcal{N}$ is a set of labelled sentences called the *main sentences* of ρ, while Γ specifies a set of conditions on Σ and $\prec$. each $\mathcal{D}_i$ is a set of labelled sentences, while each Γ_i is a set of conditions that have to be added cumulatively to Σ and $\prec$ after the rule is applied. The symbol "|" indicates where the branch splits. That is, an instance where a non-deterministic choice of possible outcomes has to be explored.

A rule ρ is *applicable* to a branch $(\mathcal{B}, \Sigma, \prec)$ iff $\mathcal{S}$ contains the main sentences of ρ and the conditions of Γ are satisfied. The rule **(non-empty $\mathcal{S}$)** given below has the additional condition that it is only applied when no other rules are applicable.

We also require that *applicable rules* have not already been satisfied. That is, that the denominators have not occurred in the branch before, and in the case of "fresh" labels in the denominator, that there are no existing labels $n \in \mathbb{N}$ which satisfy all the conditions of the denominator. We write $n \in W_e$ to denote $n \in \Sigma(e)$ and define $W^{\phi}_{\mathcal{B}} := \{n \in \mathbb{N} \mid n :: \phi \in \mathcal{B}\}$, where $\mathcal{B}$ is a branch in the tableau. $n \in \min_{\prec} X$ denotes that n is a minimal element of the set X. That is, $n' \in X$ implies $n' \not\prec n$. We also use $n^\star$ to denote the addition of a "fresh" label which has not been used before in the tableau. Our tableau rules are defined in Figure 2.

Many of the rules are straightforward, or have been discussed in the literature [3], but we provide an explanation for some of the rules which are specific to our case. In Section 2. of Figure 2, the rules $(\cap)$ and $(\cup)$ allocate every label n within a complex standpoint into the possible set of standpoint literals associated with this complex standpoint literals. This is important to ensure termination, and in order for the modal rules to be applied to each label correctly, since the conditions in the numerators of modal rules are written specifically in terms of atomic standpoints. Rules $(*_1)$ and $(*_2)$ make sure every label is associated to the universal standpoint. Rule $(\bot_-)$ accounts for the semantic contradiction which occurs when a label is allocated to both a standpoint and its negation. Rules $(\Box_c)$ and $(\Box_c^+)$ deal with sentences bound by strict modalities with conjuncts as indexes, and the conditions are phrased so that the applicability of the rule applies to labels which fulfil the required atomic standpoints. $(\Box_c^+)$ deals with conjuncts with no negative literals, while $(\Box_c)$ deals with conjuncts with negative literals. The

1. Classical Rules:

$$(\bot)\ \frac{n :: \alpha,\ n :: \neg\alpha}{n :: \bot} \quad (\neg)\ \frac{n :: \neg\neg\alpha}{n :: \alpha} \quad (\wedge)\ \frac{n :: \alpha \wedge \beta}{n :: \alpha,\ n :: \beta} \quad (\vee)\ \frac{n :: \neg(\alpha \wedge \beta)}{n :: \neg\alpha \mid n :: \neg\beta}$$

2. Standpoint Hierarchy and Modality Rules:

$$(\cap)\frac{n \in W_{e\cap d}}{n \in W_e, n \in W_d} \quad (\cup)\frac{n \in W_{e\cup d}}{n \in W_e | n \in W_d} \quad (\bot_-)\frac{n \in W_e, n \in W_{-e}}{n :: \bot}$$

$$(*_1)\frac{n :: \alpha}{n \in W_*} \quad (*_2)\frac{n \in W_e}{n \in W_*} \quad (\Diamond_e)\frac{n :: \neg\Box_e\alpha}{n^\star :: \neg\alpha; n^\star \in W_e}$$

$$(\Box_{e\cup d})\frac{n :: \Box_{e\cup d}\alpha}{n :: \Box_e\alpha, n :: \Box_d\alpha} \quad (\Box_c)\frac{n :: \Box_{s_1\cap s_2 \ldots \cap s_k \cap -s_{k+1} \cap \ldots \cap -s_m}\alpha; n' \in \Gamma^c}{n' \in W_{s_{k+1}\cup \ldots \cup s_m} | n' :: \alpha}$$

$$(\Box_{c+})\frac{n :: \Box_{s_1\cap s_2 \ldots \cap s_k}\alpha; n' \in \Gamma^c}{n' :: \alpha} \quad \text{where } \Gamma^c = \{n' \in \mathbb{N} \mid n' \in W_{s_1}, ..., n' \in W_{s_k}\}.$$

3. Defeasibility and Minimality Rules:

$$(\widetilde{\Diamond}_e)\frac{n :: \neg \widetilde{\Box}_e\ \alpha}{n^\star :: \neg\alpha; n^\star \in \min_\prec W_e} \quad (\leadsto)\frac{n :: \alpha \leadsto \beta}{n :: \neg\alpha | n^\star :: \alpha; n^\star \prec n | n :: \beta}$$

$$(\not\leadsto)\frac{n :: \neg(\alpha \leadsto \beta)}{n :: \alpha, n :: \neg\beta; n \in \min_\prec W^\alpha_{\mathcal{B}}}$$

$$(\bot_\prec)\frac{n \in \min_\prec W, n' \prec n, n' \in W}{n :: \bot} \quad (\lesssim)\frac{n :: c_1 \cup \ldots \cup c_m \lesssim d; n' \in \Gamma}{n' \in W_{s_{k+1}\cup \ldots \cup s_l} | n' \in W_d | n^\star \in W_d, \Gamma^\star}$$

$$(\lesssim^+)\frac{n :: c_1 \cup \ldots \cup c_m \lesssim d; n' \in \Gamma^+}{n' \in W_d | n^\star \in W_d, \Gamma^\star} \quad (\not\lesssim)\frac{n :: \neg(e \lesssim d)}{n^\star \in \min_\prec W_e, n^\star \in W_{-d}}$$

$$(\widetilde{\Box})\frac{n :: \widetilde{\Box}_{c_1\cup \ldots \cup c_m}\ \alpha; n' \in \Gamma}{n' :: \alpha | n' \in W_{s_{k+1}\cup \ldots \cup s_l} | n^\star :: \alpha; \Gamma^\star} \quad (\widetilde{\Box}^+)\frac{n :: \widetilde{\Box}_{c_1\cup \ldots \cup c_m}\ \alpha; n' \in \Gamma^+}{n' :: \alpha | n^\star :: \alpha; \Gamma^\star}.$$

where $\Gamma^+ = \{n' \in \mathbb{N} \mid n' \in W_{s_1}, ..., n' \in W_{s_k}, c_i = s_1 \cap \ldots \cap s_k \text{ for some } 1 \leq i \leq m\}$,
$\Gamma^\star = \{n^\star \prec n', n^\star \in W_{c_1\cup \ldots \cup c_m}\}$, and
$\Gamma = \{n' \in \mathbb{N} \mid n' \in W_{s_1}, ..., n' \in W_{s_k}, c_i = s_1 \cap \ldots \cap s_k \cap -s_{k+1} \cap \ldots \cap -s_l \text{ where } 1 \leq i \leq m\}$

(*non-empty* $\mathcal{S}$):
If, after all other applicable rules are applied to a branch $(\mathcal{B}, \Sigma, \prec)$, there is some $s \in \mathcal{S}$ such that $n \in W_s$ does not appear for any $n \in \mathbb{N}$, then add $n^\star \in W_s$ to Σ.

Fig. 2. Tableau Rules for Local Satisfiability in $\widetilde{\mathcal{L}}_{\mathbb{S}}$

first branch in the denominator of $(\Box_c)$ intuitively accounts for the case where a label n satisfies the positive literals in the conjunct but is not in one of the negative conjuncts, and so the sentence $n :: \alpha$ need not appear. Rule $(\Box_{e\cup d})$ deals with indexes which have unions of multiple conjuncts. Section 3. describes the behaviour of defeasible parts of the logic. Rules $(\approxeq_e)$, $(\leadsto)$, $(\not\leadsto)$ and $(\bot_{\prec})$ follow similar intuitions to those appearing in tableau construted by Britz and Varzicnzak [3]. Rules $(\lesssim)$ and $(\lesssim^+)$ allocates necessary labels to a new standpoint when a defeasible sharpening occurs. The conditions are again referred to in terms of atomic standpoints, and rules $(\lesssim)$ and $(\lesssim^+)$ differ in the same manner that $(\Box_c)$ and $(\Box_c^+)$ do. Both rules also have a branch which expresses that when $n :: e \lesssim d$ occurs, it is possible that a label in W_e is not allocated to W_d on the account of it being non-minimal. Rule $(\not\lesssim)$ deals with negated defeasible sharpenings. Lastly, rules $(\approxeq)$ and $(\approxeq^+)$ deal with sentences bound by $\approxeq$ with a similar differentiation between conjuncts with literals and without. It is worth noting that by Proposition 2, we cannot break unions of conjuncts down into their parts as is done in classical modalities, and so have to treat the general case of sentences in INF.

Definition 12. *[3] A **tableau** $\mathcal{T}$ for $\alpha \in \widetilde{\mathcal{L}}_{\mathbb{S}}$ is the limit of a sequence $\mathcal{T}^0, ..., \mathcal{T}^k, ...$ of sets of branches where the initial $\mathcal{T}^0 := \{(\{0 :: \alpha\}, \emptyset, \emptyset)\}$ and every $\mathcal{T}^{i+1}$ is obtained by the application of one of the applicable rules in Figure 2 to some branch in $\mathcal{T}^i$. Such a limit is denoted $\mathcal{T}^\infty$.*

We assume here that the limit is only found once every applicable rule is applied. We say a tableau is *saturated* if no rule is applicable to any of its branches.

Definition 13. *A branch $(\mathcal{B}, \Sigma, \prec)$ is **closed** iff $n :: \bot \in \mathcal{B}$ for some n. A saturated tableau $\mathcal{T}$ for $\alpha \in \widetilde{\mathcal{L}}_{\mathbb{S}}$ is closed iff all its branches are closed. If a saturated tableau $\mathcal{T}$ is not closed, we say that it is **open**.*

We then can describe the *tableau algorithm* for local satisfaction checking as follows: If we are given a sentence $\alpha \in \widetilde{\mathcal{L}}_{\mathbb{S}}$ and want to check if it is locally satisfiable, then we construct a saturated tableau for α as in Definition 12. If the resulting tableau is closed, then we conclude that α *is not* locally satisfiable, and if the tableau is open, we conclude that α *is* locally satisfiable. However, in order for this algorithm to be useful, we need the proposed tableau calculus to be sound, complete and to terminate. We therefore present the following two theorems, which are the main results of our paper:

Theorem 1 (Complexity). *The tableau algorithm runs in* PSPACE.

This tells us that our tableau calculus terminates and that it is in the same complexity class as the tableau algorithms for modal logic $\mathbf{K}_n$ with defeasible modalities [2], as well as for the classical normal modal logics $\mathbf{K}$ and $\mathbf{K}_n$ [13, 15].

Theorem 2 (Soundness and Completeness.). *The tableau algorithm is sound and complete with respect to local satisfiability in SPSS semantics.*

Moreover, by Theorem 2 and Proposition 5, we can easily adapt our tableau algorithm in order to obtain algorithms for global satisfiability and for preferential entailment which are sound, complete and computable in PSPACE. Lastly, it is worth noting that the rules in Sections 1 and 2 in Figure 2 provide a tableau calculus for an extension of Gómez Álvarez and Rudolph's [7] classical propositional standpoint logic, in which full complex standpoint expressions are permitted. This is on its own a novel contribution to the field of standpoint logics.

5 Related Work

The most closely related work to this paper is that of Leisegang et al. [18] who also consider combining defeasibility and standpoint logics. However, their paper aims at representing situations where standpoints *hold* defeasible beliefs, while this paper considers defeasibility and typicality relations between the precisifications themselves. In particular, the language DRSL is given in the form $\phi ::= \psi \mid \phi \wedge \phi \mid \Box_s \psi \mid \Diamond_s \psi$,[3] where ψ is a boolean formula or a defeasible implication $\alpha \rightsquigarrow \beta$ where α and β are Boolean. The semantics is given by *ranked standpoint structures*, which consist of a triple $M = (\Pi, \sigma, \gamma)$ where Π and σ are as in classical standpoint structures. However, γ maps each precisification not to a classical valuation, but a *ranked interpretation* as defined by Lehmann and Magidor [17]. That is, each $\gamma(\pi)$ is a ranking function $\gamma(\pi) : 2^{\mathcal{P}} \to \mathbb{N} \cup \{\infty\}$. Such a ranking function intuitively expresses how "typical" or preferred a state of the world is, with lower rankings signifying more typical states. This in turn induces a preference ordering for each precisification where $v \prec_\pi u$ iff $\gamma(\pi)(v) < \gamma(\pi)(u)$. Then if α and β are Boolean formulae, $M, \pi \Vdash \alpha$ iff $\gamma(v) \neq \infty$ implies $v \Vdash \alpha$ $M, \pi \Vdash \alpha \rightsquigarrow \beta$ iff $\min_{\prec_\pi} [\![\alpha]\!] \subseteq [\![\beta]\!]$. The satisfaction standpoint modalities and conjunctions are defined inductively on top of this as in the classical case (for example, $M, \pi \Vdash \Box_s \phi$ iff $M, \pi' \Vdash \phi$ for all $\phi \in \sigma(s)$). These semantics for DRSL therefore utilize preference orderings, but such orderings are internal to the underlying valuation of each precisification, while the ordering in PDSL occurs on the set of precisifications itself. Another distinction in the work of Leisegang et al. [18] is that it focuses on extending *rational closure*, a non-monotonic form of reasoning introduced by Lehman and Magidor [17], into the DRSL case, while our work focusses on an extension of preferential entailment.

Besides this work, defeasibility has been considered for basic normal modal logics by Britz et al. [1, 2] and Britz and Varzinczak [3], and in the case of Linear Temporal Logic by Chafik et al. [5]. Standpoint modalities have been considered in the propositional case by Gómez Álvarez and Rudolph [7], in first-order logic and its decidable fragments by Gómez Álvarez et al. [8–10] and in the case of linear temporal logic by Gigante et al. [6], who also use semantic tableau methods. Other forms of non-monotonic reasoning in standpoint logics have been considered by Gorczyca and Straß [11].

6 Conclusion

In this paper, we propose an extension to both defeasible modalities, and standpoint logics by considering a logic of defeasible standpoint modalities. We define the language $\widetilde{\mathcal{L}}_{\mathbb{S}}$ which extends propositional standpoint logic with defeasible modalities of the form

[3] As well as classical sharpening statements.

$\widetilde{\Box}_e$ and $\widetilde{\Diamond}_e$, as well as defeasible standpoint sharpenings and implications. The main contribution of the paper is to provide a semantics for the logical language in Section 3, and provide a sound, complete and terminating method to check satisfiability with respect to these semantics. In particular, in Section 4 we consider a translation to plain standpoint logic which allows for an NP-complete satisfiability checking in a restricted setting, and go on to provide a tableau algorithm for the unrestricted case which is computable in PSPACE.

For future work, we believe investigating other forms of non-monotonic entailment for defeasible reasoning in the propositional case [16, 17] in the language $\widetilde{\mathcal{L}}_{\mathbb{S}}$ would expand the given logic's ability to reason prototypically about information. Moreover, it would be worth investigating whether defeasible standpoint modalities can be added to more expressive logics, such as lightweight description logics, where classical standpoint modalities have been investigated [9, 10]. Lastly, it would be worth investigating a deeper comparison and fusion of the logic proposed in our paper with the related approach to defeasibility in standpoint logics given by Leisegang et al. [18].

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