Introducing Role Defeasibility in Description Logics

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Abstract. Accounts of preferential reasoning in Description Logics often take as point of departure the semantic notion of a preference order on objects in a domain of interpretation, which allows for the development of notions of defeasible subsumption and entailment. However, such an approach does not account for defeasible roles, interpreted as partially ordered sets of tuples. We state the case for role defeasibility and introduce a corresponding preferential semantics for a number of defeasible constructs on roles. We show that this does not negatively affect decidability or complexity of reasoning for an important class of DLs, and that existing notions of preferential reasoning can be expressed in terms of defeasible roles.

Keywords: Description Logics, defeasible reasoning, preferential semantics

1 Introduction

Description Logics (DLs) [2] are a family of logic-based knowledge representation formalisms with appealing computational properties and a variety of applications at the confluence of modern artificial intelligence and other areas. In this regard, endowing DLs and their associated reasoning services with the ability to cope with defeasibility is a natural step in their development. Indeed, the past two decades have witnessed the surge of many attempts to introduce non-monotonic reasoning capabilities in a DL setting. These range from preferential approaches [14, 15, 20, 22, 25, 27, 39, 40] to circumscription-based ones [6, 7, 41], amongst others [3, 4, 23, 29–31, 37, 38, 43].

Given the special status of subsumption in DLs in particular and the historical importance of entailment in logic in general, the bulk of the effort in this direction has quite naturally been put in the definition of a proper account of defeasible subsumption and in the characterisation of appropriate notions of defeasible entailment relations. Semantically, in the latter, orderings on the class of first-order interpretations are usually considered [7, 12, 27, 28, 39], whereas in the former, a typicality ordering on the objects of the domain of interpretation is put forward [14, 15, 26, 25].

Here we investigate a complementary notion, namely that of relativised role defeasibility. Our motivation stems essentially from the observation that a given relationship holding between some objects may be deemed more normal than between others, and that this may be the case irrespective of whether the relevant objects are typical in one way or another. As an example, consider the role name $\mathsf{guardianOf}$: 'Normal' tuples in its extension (the relation it is interpreted as) may be guardian-ward tuples where the ward is a minor and the guardian a parent or natural guardian, while an 'exceptional' tuple may be a guardian-ward tuple where the ward is an adult with an appointed legal guardian. In this example, there is nothing exceptional about either the legal guardian or the ward — the exceptionality rather lies in the nature of their relationship. The role name therefore provides a primitive context relative to which exceptionality is determined, while exceptionality is evaluated semantically by comparing tuples in the role extension.

As a semantic means to capture the nuances of normality at the level of roles as motivated above, in this work we propose placing a parameterised preference order on binary relations over the domain. Armed with the semantic constructions we shall define and study here, we will see that it becomes possible to:

• Define *plausible value restrictions* [13] of the form $\forall\!\!\!\forall r.C$, as in $\forall\!\!\!\forall \mathsf{guardianOf}.\mathsf{Minor}$, which intuitively refers to those individuals whose normal guardianship relations are of minors, whilst being, for instance, the legal guardian of a developmentally disabled adult;

• Define *plausible (qualified) number restrictions* of the form $\gtrsim nr.C$ or $\lesssim nr.C$ (or $\approx nr.C$), as in $\lesssim 2\mathsf{hasSibling}.\mathsf{Female}$, referring to the individuals in at most two normal sibling relationships with sisters (but who can still have a stepsister), or even $\approx 1\mathsf{marriedTo}.\top$, which describes the individuals in one normal marriage (but who can nevertheless be in a type of wedlock with someone else);

• State *plausible role inclusions* of the form $r_1 \sqsubsetsim r_2$, as in $\mathsf{parentOf} \sqsubsetsim \mathsf{progenitorOf}$, stipulating that the role of being a parent is usually (but not necessarily) that of also being the progenitor;

• State *role-typicality axioms* of the form $\star r_1 \sqsubseteq \star r_2$ and $\star r(a,b)$, where $\star$ is an extension of typicality operators [9, 10, 25, 27] that we shall define for role names (and, more generally, for compound roles). For example, $\star\mathsf{progenitorOf} \sqsubseteq \star\mathsf{hasChild}$ says that typical procreation implies typical parenthood, while the assertion $\star\mathsf{hasChild}(\mathsf{john},\mathsf{anne})$ conveys the information that the tuple $(\mathsf{john},\mathsf{anne})$ is to be regarded as a typical one in the interpretation of role $\mathsf{hasChild}$;

• State *plausible role disjointness* of the form $\star r_1 \sqcap \star r_2 \sqsubseteq \bot$, as for instance in $\star\mathsf{hasSibling} \sqcap \star\mathsf{marriedTo} \sqsubseteq \bot$, the meaning of which speaks for itself;

• State *plausible role characteristics*, for instance saying that role $\mathsf{marriedTo}$ is *normally functional* and that partOf is *usually transitive*, while still allowing for exceptions, i.e., for exceptional tuples to fail the relation's property under consideration, thereby not ruling out, in the former example, the existence of polygamous mariages.

Moreover, we shall see that, with our enriched semantics, it also becomes possible to provide an alternative account of *plausible concept subsumptions* [14, 15, 22, 26] of the form $C \sqsubsetsim D$, as for instance in $\mathsf{Mother} \sqsubsetsim \exists\mathsf{hasPartner}.\top$, of which the intuition is that, usually, mothers have a partner.

By putting all of that into place, we hope to open up an avenue for further explorations of defeasibility in Description Logics, in particular in extensions of the preferential approach therein.

In the remainder of the present paper, we take the following route: after presenting the required background on DLs (Section 2), we introduce the semantic construction the core of the paper builds upon (Section 3). We then move on by studying new defeasible constructs capturing several aspects of role defeasibility (Section 4). In Section 5, we show that the important notion of plausible concept subsumption can be embedded within our framework. We then conclude with some remarks on related work and possible strands for future investigation.

2 The Description Logic $\mathcal{ALC}$

The (concept) language of $\mathcal{ALC}$ is built upon a finite set of atomic *concept names* $\mathsf{N}_{\mathscr{C}}$, a finite set of *role names* $\mathsf{N}_{\mathscr{R}}$ and a finite set of *individual names* $\mathsf{N}_{\mathscr{I}}$ such that $\mathsf{N}_{\mathscr{C}}$, $\mathsf{N}_{\mathscr{R}}$ and $\mathsf{N}_{\mathscr{I}}$ are pairwise disjoint. With $A, B, \ldots$ we denote atomic concepts, with $r, s, \ldots$ role names, and with $a, b, \ldots$ individual names. Complex concepts are denoted with $C, D, \ldots$ and are built according to the rule:

$$C ::= \top \mid \bot \mid A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \forall r.C \mid \exists r.C$$

With $\mathcal{L}_{\mathcal{ALC}}$ we denote the *language* of all $\mathcal{ALC}$ concepts.

The semantics of $\mathcal{L}_{\mathcal{ALC}}$ is the standard set theoretic Tarskian semantics. An *interpretation* is a structure $\mathcal{I} := \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, where $\Delta^{\mathcal{I}}$ is a non-empty set called the *domain*, and $\cdot^{\mathcal{I}}$ is an *interpretation function* mapping concept names A to subsets $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$, role names r to binary relations $r^{\mathcal{I}}$ over $\Delta^{\mathcal{I}}$, and individual names a to elements of the domain $\Delta^{\mathcal{I}}$, i.e., $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$.

As an example, let $\mathsf{N}_{\mathscr{C}} := \{A_1, A_2, A_3\}$, $\mathsf{N}_{\mathscr{R}} := \{r_1, r_2\}$ and $\mathsf{N}_{\mathscr{I}} := \{a_1, a_2, a_3\}$. Figure 1 depicts the DL interpretation $\mathcal{I}_1 = \langle \Delta^{\mathcal{I}_1}, \cdot^{\mathcal{I}_1} \rangle$, where $\Delta^{\mathcal{I}_1} = \{x_i \mid 1 \leq i \leq 9\}$, $A_1^{\mathcal{I}_1} = \{x_1, x_4, x_6\}$, $A_2^{\mathcal{I}_1} = \{x_3, x_5, x_9\}$, $A_3^{\mathcal{I}_1} = \{x_6, x_7, x_8\}$, $r_1^{\mathcal{I}_1} = \{(x_1, x_6), (x_4, x_8), (x_2, x_5)\}$, $r_2^{\mathcal{I}_1} = \{(x_4, x_4), (x_6, x_4), (x_5, x_8), (x_9, x_3)\}$, $a_1^{\mathcal{I}_1} = x_5$, $a_2^{\mathcal{I}_1} = x_1$, $a_3^{\mathcal{I}_1} = x_2$.

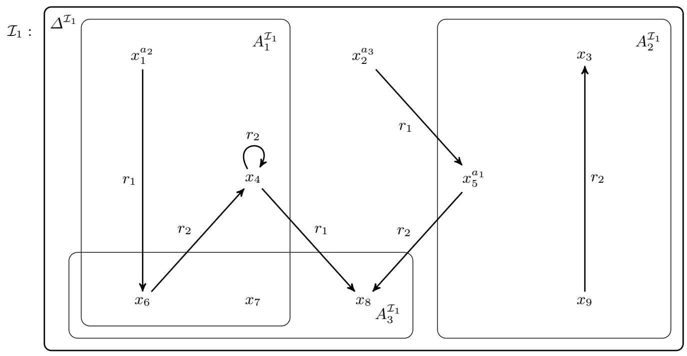

Fig. 1. An interpretation for $\mathsf{N}_{\mathscr{C}} = \{A_1, A_2, A_3\}$, $\mathsf{N}_{\mathscr{R}} = \{r_1, r_2\}$ and $\mathsf{N}_{\mathscr{I}} = \{a_1, a_2, a_3\}$.

Given an interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, $\cdot^{\mathcal{I}}$ is extended to interpret complex concepts of $\mathcal{L}_{\mathcal{ALC}}$ in the following way:

$$\top^{\mathcal{I}} := \Delta^{\mathcal{I}}, \quad \bot^{\mathcal{I}} := \emptyset, \quad (\neg C)^{\mathcal{I}} := \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$$

$$(C \sqcap D)^{\mathcal{I}} := C^{\mathcal{I}} \cap D^{\mathcal{I}}, \quad (C \sqcup D)^{\mathcal{I}} := C^{\mathcal{I}} \cup D^{\mathcal{I}}$$

$$(\exists r.C)^{\mathcal{I}} := \{x \in \Delta^{\mathcal{I}} \mid r^{\mathcal{I}}(x) \cap C^{\mathcal{I}} \neq \emptyset\}, \quad (\forall r.C)^{\mathcal{I}} := \{x \in \Delta^{\mathcal{I}} \mid r^{\mathcal{I}}(x) \subseteq C^{\mathcal{I}}\}$$

As an example, in the interpretation $\mathcal{I}_1$, we have $(A_1 \sqcup A_3)^{\mathcal{I}_1} = \{x_1, x_4, x_6, x_7, x_8\}$, $(A_1 \sqcap A_3)^{\mathcal{I}} = \{x_6, x_7\}$, $(\exists r_1.A_3)^{\mathcal{I}_1} = \{x_1, x_4\}$ and $(\forall r_2.A_2)^{\mathcal{I}} = \{x_9\}$.

Given $C, D \in \mathcal{L}_{\mathcal{ALC}}$, $C \sqsubseteq D$ is a *subsumption statement*, read "C is subsumed by D" (or, alternatively, "D is more general than C" or "C is more specific than D"). $C \equiv D$ is an abbreviation for both $C \sqsubseteq D$ and $D \sqsubseteq C$. An $\mathcal{ALC}$ *TBox* $\mathcal{T}$ is a finite set of subsumption statements and formalises the *intensional* knowledge about a given domain of application. Given $C \in \mathcal{L}_{\mathcal{ALC}}$, $r \in \mathsf{N}_{\mathscr{R}}$ and $a, b \in \mathsf{N}_{\mathscr{I}}$, an *assertional statement* (*assertion*, for short) is an expression of the form $C(a)$ or $r(a, b)$. An $\mathcal{ALC}$ *ABox* $\mathcal{A}$ is a finite set of assertional statements formalising the *extensional* knowledge of the domain. We shall denote statements, both subsumption and assertional, with $\alpha, \beta, \ldots$. Given $\mathcal{T}$ and $\mathcal{A}$, with $\mathcal{K} := \mathcal{T} \cup \mathcal{A}$ we denote an $\mathcal{ALC}$ *knowledge base*.

An interpretation $\mathcal{I}$ *satisfies* a subsumption statement $C \sqsubseteq D$ (denoted $\mathcal{I} \Vdash C \sqsubseteq D$) if and only if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$. (And then $\mathcal{I} \Vdash C \equiv D$ if and only if $C^{\mathcal{I}} = D^{\mathcal{I}}$.) In the example of Figure 1, we have $\mathcal{I}_1 \Vdash \exists r_1.A_3 \sqsubseteq A_1$ and $\mathcal{I}_1 \nVdash A_1 \sqcap A_3 \sqsubseteq \forall r_2.A_2$. An interpretation $\mathcal{I}$ *satisfies* an assertion $C(a)$ (respectively, $r(a, b)$), denoted $\mathcal{I} \Vdash C(a)$ (respectively, $\mathcal{I} \Vdash r(a, b)$), if and only if $a^{\mathcal{I}} \in C^{\mathcal{I}}$ (respectively, $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$). In the above example, we have both $\mathcal{I}_1 \Vdash A_1 \sqcap \neg A_3(a_2)$ and $\mathcal{I}_1 \Vdash r_1(a_3, a_1)$, but $\mathcal{I}_1 \nVdash \forall r_1.A_2(a_2)$.

We say that an interpretation $\mathcal{I}$ is a *model* of a TBox $\mathcal{T}$ (denoted $\mathcal{I} \Vdash \mathcal{T}$) if and only if $\mathcal{I} \Vdash \alpha$ for every $\alpha \in \mathcal{T}$. Analogously, $\mathcal{I}$ is a model of an ABox $\mathcal{A}$ (denoted $\mathcal{I} \Vdash \mathcal{A}$) if and only if $\mathcal{I} \Vdash \alpha$ for every $\alpha \in \mathcal{A}$. We say that $\mathcal{I}$ is a model of a knowledge base $\mathcal{K} = \mathcal{T} \cup \mathcal{A}$ if and only if $\mathcal{I} \Vdash \mathcal{T}$ and $\mathcal{I} \Vdash \mathcal{A}$. A statement α is (classically) *entailed* by a knowledge base $\mathcal{K}$, denoted $\mathcal{K} \models \alpha$, if and only if every model of $\mathcal{K}$ satisfies α. If $\mathcal{K} = \emptyset$, then we have that $\mathcal{I} \Vdash \alpha$ for all interpretations $\mathcal{I}$, in which case we say α is a *validity* and denote with $\models \alpha$.

For more details on Description Logics in general and on $\mathcal{ALC}$ in particular, the reader is invited to consult the Description Logic handbook [2].

3 r-Ordered Interpretations

We now formalise the intuitive notions we briefly presented in the Introduction. Given a DL interpretation $\mathcal{I}$, we enrich it with a collection of preference relations, one for (the interpretation of) each role name in $\mathsf{N}_{\mathscr{R}}$.

Definition 1 (r-Ordered Interpretation). *An* r-ordered interpretation *is a tuple* $\mathcal{R} := \langle \Delta^{\mathcal{R}}, \cdot^{\mathcal{R}}, \prec^{\mathcal{R}} \rangle$ *in which* $\langle \Delta^{\mathcal{R}}, \cdot^{\mathcal{R}} \rangle$ *is a (classical) DL interpretation (see Section 2), and* $\prec^{\mathcal{R}} := \langle \prec_1^{\mathcal{R}}, \ldots, \prec_n^{\mathcal{R}} \rangle$*, where each* $\prec_i^{\mathcal{R}} \subseteq r_i^{\mathcal{R}} \times r_i^{\mathcal{R}}$*, for* $1 \leq i \leq n$*, is a well-founded strict partial order on* $r_i^{\mathcal{R}}$*, i.e., each* $\prec_i^{\mathcal{R}}$ *is irreflexive, transitive and every non-empty* $R \subseteq r_i^{\mathcal{R}}$ *has minimal elements w.r.t.* $\prec_i^{\mathcal{R}}$ *(see Definition 2 below).*

As an example, let $\mathsf{N}_{\mathscr{C}} := \{A_1, A_2, A_3\}$, $\mathsf{N}_{\mathscr{R}} := \{r_1, r_2\}$, $\mathsf{N}_{\mathscr{I}} := \{a_1, a_2, a_3\}$, and let the r-ordered interpretation $\mathcal{R}_1 = \langle \Delta^{\mathcal{R}_1}, \cdot^{\mathcal{R}_1}, \prec^{\mathcal{R}_1} \rangle$, where $\Delta^{\mathcal{R}_1} = \Delta^{\mathcal{I}_1}$, $\cdot^{\mathcal{R}_1} = \cdot^{\mathcal{I}_1}$, and $\prec^{\mathcal{R}_1} = \langle \prec_1^{\mathcal{R}_1}, \prec_2^{\mathcal{R}_1} \rangle$, where $\prec_1^{\mathcal{R}_1} = \{(x_4x_8, x_2x_5), (x_2x_5, x_1x_6), (x_4x_8, x_1x_6)\}$ and $\prec_2^{\mathcal{R}_1} = \{(x_6x_4, x_4x_4), (x_5x_8, x_9x_3)\}$. (For the sake of readability, we shall henceforth sometimes write tuples of the form (x, y) as xy.) Figure 2 below depicts the r-ordered interpretation $\mathcal{R}_1$. In the picture, $\prec_1^{\mathcal{R}_1}$ and $\prec_2^{\mathcal{R}_1}$ are represented, respectively, by the dashed and the dotted arrows. (Note the direction of the $\prec^{\mathcal{R}}$-arrows, which point from more preferred to less preferred pairs of objects.) Also for the sake of readability, we shall omit the transitive $\prec^{\mathcal{R}}$-arrows.

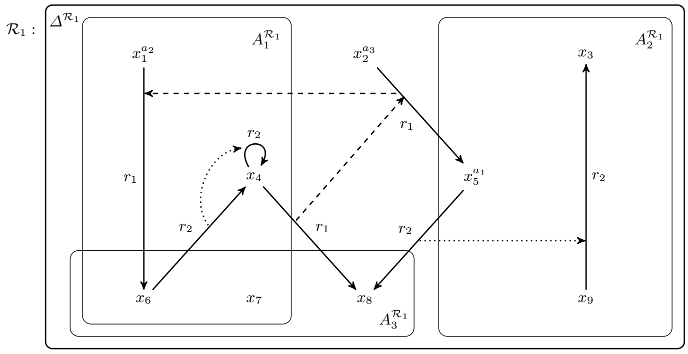

Fig. 2. An r-ordered interpretation for $\mathsf{N}_{\mathscr{C}}$, $\mathsf{N}_{\mathscr{R}}$ and $\mathsf{N}_{\mathscr{I}}$ as in Figure 1.

Given $\mathcal{R} = \langle \Delta^{\mathcal{R}}, \cdot^{\mathcal{R}}, \prec^{\mathcal{R}} \rangle$, the intuition of $\Delta^{\mathcal{R}}$ and $\cdot^{\mathcal{R}}$ is the same as in a standard DL interpretation. The intuition underlying each of the orderings in $\prec^{\mathcal{R}}$ is that they play the role of *preference relations* (or *normality orderings*), in a sense similar to that introduced by Shoham [42] with a preference on worlds in a propositional setting and as extensively investigated by Kraus et al. [32, 33] and others [11, 14, 25]: the pairs (x, y) that are lower down in the ordering $\prec_i^{\mathcal{R}}$ are deemed as the most normal (or typical, or expected) in the context of (the interpretation of) r_i. Technically, the difference between our definitions and those in the aforementioned work lies on the fact that our $\prec_i^{\mathcal{R}}$ are orderings on binary relations on the domain $\Delta^{\mathcal{R}}$, instead of orderings on propositional valuations or on plain objects of $\Delta^{\mathcal{R}}$.

It is worth spelling out that we do not require that pairs of objects intrinsically possess certain features that render some of them more normal than others. Rather, the intention is to provide a framework in which to express all conceivable ways in which such pairs can be ordered, in the same way that the class of all classical DL interpretations constitute a framework representing all conceivable (logically allowed) ways of representing the properties of objects and their relationships with other objects. It is up to the knowledge base at hand to impose constraints on the allowed orderings on pairs of objects in r-ordered DL interpretations in the same way as it imposes constraints on the allowed extensions of classes and roles in standard DL interpretations. (This point will become more clear from Section 4 onwards.)

Definition 2 (Minimality w.r.t. $\prec_i^{\mathcal{R}}$). *Let $\mathcal{R} = \langle \Delta^{\mathcal{R}}, \cdot^{\mathcal{R}}, \prec^{\mathcal{R}} \rangle$ be an r-ordered interpretation and let $R \subseteq r_i^{\mathcal{R}}$, for some $1 \leq i \leq n$. Then $\min_{\prec_i^{\mathcal{R}}} R := \{(x, y) \in R \mid$ there is no $(x', y') \in R$ such that $(x', y') \prec_i^{\mathcal{R}} (x, y)\}$, i.e., $\min_{\prec_i^{\mathcal{R}}} R$ denotes the minimal elements of R w.r.t. the preference relation $\prec_i^{\mathcal{R}}$ associated to $r_i^{\mathcal{R}}$.*

Since we assume each $\prec_i^{\mathcal{R}}$ to be a well-founded strict partial order on the respective $r_i^{\mathcal{R}}$, we are guaranteed that for every $R \subseteq r_i^{\mathcal{R}}$ such that $R \neq \emptyset$, $\min_{\prec_i^{\mathcal{R}}} R$ is well defined. (The reader familiar with the KLM approach [32] will immediately see that this implies a version of the *smoothness condition* for pairs of objects.) As an example, in Figure 2, $\min_{\prec_1^{\mathcal{R}_1}} r_1^{\mathcal{R}_1} = \{x_4 x_8\}$.

An r-ordered interpretation $\mathcal{R}$ satisfies a (classical) subsumption statement $C \sqsubseteq D$ (denoted $\mathcal{R} \Vdash C \sqsubseteq D$) if and only if $C^{\mathcal{R}} \subseteq D^{\mathcal{R}}$. It satisfies an assertion $C(a)$ (respectively, $r(a,b)$), denoted $\mathcal{R} \Vdash C(a)$ (respectively, $\mathcal{R} \Vdash r(a,b)$), if and only if $a^{\mathcal{R}} \in C^{\mathcal{R}}$ (respectively, $(a^{\mathcal{R}}, b^{\mathcal{R}}) \in r^{\mathcal{R}}$). It is easy to see that the addition of the $\prec^{\mathcal{R}}$-component preserves the truth of all classical statements holding in the remaining structure. That is, if $\mathcal{R} = \langle \Delta^{\mathcal{R}}, \cdot^{\mathcal{R}}, \prec^{\mathcal{R}} \rangle$, then for every α, $\mathcal{R} \Vdash \alpha$ if and only if $\langle \Delta^{\mathcal{R}}, \cdot^{\mathcal{R}} \rangle \Vdash \alpha$. The role of the $\prec^{\mathcal{R}}$-components will become patent in the next section.

4 Role-Plausibility Constructs

In this section, we present the defeasible role constructs promised in the Introduction. Before doing so, we recall some distinguishing properties of general operators for defeasible reasoning, against which we shall check each of the operators to be introduced in the sequel: Given $n+1$ partially ordered sets of objects $\langle S_i, \leq_i \rangle$, $0 \leq i \leq n$, an n-ary function $f : \Pi_{i=0}^{n-1} S_i \to S_n$ is:

- *monotone (increasing)* on S_n if the following holds:
  If $x_i \leq y_i$ for $0 \leq i < n$, then $f(x_0, \ldots, x_{n-1}) \leq f(y_0, \ldots, y_{n-1})$;
- *ampliative* with respect to an n-ary function $h : \Pi_{i=0}^{n-1} S_i \to S_n$ if:
  $h(x_1, \ldots, x_n) \leq f(x_1, \ldots, x_n)$, for all $x_i \in S_i, 0 \leq i < n$;
- *strictly ampliative* with respect to h if it is ampliative w.r.t. h, and also:
  $h(x_1, \ldots, x_n) < f(x_1, \ldots, x_n)$, for some $x_i \in S_i, 0 \leq i < n$.

A function is *non-monotonic* if it is not monotone, i.e., if it fails monotonicity in at least one argument. We then observe that the concept constructor $\exists$ induces a monotone increasing function $f_\exists : \mathscr{P}(\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}) \times \mathscr{P}(\Delta^{\mathcal{I}}) \longrightarrow \mathscr{P}(\Delta^{\mathcal{I}})$, with sets ordered by set inclusion, such that $f_\exists : \langle r^{\mathcal{I}}, C^{\mathcal{I}} \rangle \mapsto (\exists r.C)^{\mathcal{I}}$. Likewise, $\forall$ induces a non-monotonic function $f_\forall : \langle r^{\mathcal{I}}, C^{\mathcal{I}} \rangle \mapsto (\forall r.C)^{\mathcal{I}}$, which is monotone in its second argument, but not in the first. We note that strict ampliativity is a necessary condition for a concept constructor to be deemed defeasible.

In the remainder of the present section, we shall use $r_i^{\mathcal{R}|x}$ as an abbreviation for $r_i^{\mathcal{R}} \cap (\{x\} \times \Delta^{\mathcal{R}})$, i.e., the restriction of the domain of $r_i^{\mathcal{R}}$ to $\{x\}$.

4.1 Plausible Value Restriction

Classical value restrictions of the form $\forall r.C$ constrain objects (in its interpretation) to those that are related by r only to objects in C. This requirement can be (and, in practice, often is) too strong. For instance, consider the concept $\forall \mathsf{guardianOf}.\mathsf{Minor}$, which we have encountered in the Introduction. An individual who has several children, but is also the legal guardian of a developmentally disabled adult would not belong to this class, even though we may want to include such an individual when referring to parents whose 'normal' guardianship role is with minors. In order to single out this case, while still being able to draw conclusions on what is typically the case about guardianship, we here make a case for *plausible* value restrictions of the form $\boldsymbol{\forall} r.C$. Intuitively, $\boldsymbol{\forall}\mathsf{guardianOf}.\mathsf{Minor}$ should cater for the example we have just seen.

Let $\mathcal{ALC}^{\boldsymbol{\forall}}$ denote $\mathcal{ALC}$ extended with plausible value restrictions. We can give $\boldsymbol{\forall}$ a natural semantics in terms of our r-ordered interpretations as follows:

$$(\boldsymbol{\forall} r_i.C)^{\mathcal{R}} := \{x \in \Delta^{\mathcal{R}} \mid \text{ for all } y \in \Delta^{\mathcal{R}}, \text{ if } (x,y) \in \min_{\prec_i^{\mathcal{R}}}(r_i^{\mathcal{R}|x}), \text{ then } y \in C^{\mathcal{R}}\}$$

Then, $\boldsymbol{\forall}$ induces a ternary function $f_{\boldsymbol{\forall}}$, with the strict partial order on the participating role as third argument, that is $f_{\boldsymbol{\forall}} : \langle r_i^{\mathcal{R}}, C^{\mathcal{R}}, \prec_{r_i}^{\mathcal{R}}\rangle \mapsto (\boldsymbol{\forall} r_i.C)^{\mathcal{R}}$.

Proposition 1. *The function $f_{\boldsymbol{\forall}}$ is non-monotonic in its first argument, monotone in its second and third arguments and is strictly ampliative w.r.t. $f_{\forall}$.*

Another useful application of plausible value restrictions is in the specification of the *normal range* of a role, as in $\top \sqsubseteq \boldsymbol{\forall} r.C$ ('the range of r is normally C'). If we allow for role inverses, we can also specify the *normal domain* of a role with $\top \sqsubseteq \boldsymbol{\forall} r^{-}.C$ ('the domain of r is normally C').

Theorem 1. *$\mathcal{ALC}^{\boldsymbol{\forall}}$ has the finite-model property and is therefore decidable.*

The proof of Theorem 1 is via the standard technique of filtration redefined for r-ordered interpretations and making sure the resulting preference relations in the filtered model are each a strict partial order on the respective role interpretation.

Theorem 2. *In $\mathcal{ALC}^{\boldsymbol{\forall}}$, concept satisfiability and subsumption w.r.t. acyclic TBoxes are* PSPACE*-complete problems. Concept satisfiability and subsumption w.r.t. general TBoxes are* EXPTIME*-complete problems.*

The lower bound follows from the lower-bound result for $\mathcal{ALC}$ alone. The proof of the upper bound is along the lines of that for classical $\mathcal{ALC}$ via automata but with an extra data structure to account for the preference relations. It can be shown that the look-up at the preference relations changes neither the time complexity (the number of nodes in the search tree remains single exponential) nor the size of each branch in the depth-first search that is carried out.

4.2 Plausible Number Restriction

Next we consider qualified number restrictions, which, in the classical case, take the form $\geq nr.C$, $\leq nr.C$ or $= nr.C$, where n is a positive integer, and which allow us to specify cardinality constraints on roles with role fillers falling under a certain concept. The classical semantics of these constructs is given by:

$$(\geq nr_i.C)^{\mathcal{I}} := \{x \in \Delta^{\mathcal{I}} \mid \#\{y \in \Delta^{\mathcal{I}} \mid (x,y) \in r_i^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\} \geq n\}$$

$$(\leq nr_i.C)^{\mathcal{I}} := \{x \in \Delta^{\mathcal{I}} \mid \#\{y \in \Delta^{\mathcal{I}} \mid (x,y) \in r_i^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\} \leq n\}$$

and $= nr.C$ is seen as an abbreviation for $(\geq nr.C) \sqcap (\leq nr.C)$. The extension of $\mathcal{ALC}$ with qualified number restrictions is called $\mathcal{ALCQ}$.

It turns out such constructs, too, can be too rigid, as the following example illustrates. The concept $\leq 2\mathsf{hasSibling.Female}$ denotes the class of people with at most two sisters and, of course, does not admit the case of individuals whose father becomes the legal guardian of a girl, thereby finding themselves with a new, unexpected sibling. In this case, we would like to be able to say that such individuals are in at most two normal sibling relationships.

To cope with cases such as this, we here introduce plausible versions of qualified number restrictions of the form $\gtrsim nr.C$, $\lesssim nr.C$ (and $\approx nr.C$). Let $\mathcal{ALCQ}^{\gtrsim}$ denote $\mathcal{ALCQ}$ extended with plausible number restrictions. These new concept constructors can be given a semantics in terms of our r-ordered interpretations in the following way:

$$(\gtrsim nr_i.C)^{\mathcal{R}} := \{x \in \Delta^{\mathcal{R}} \mid \#\{y \in \Delta^{\mathcal{R}} \mid (x,y) \in \min_{\prec_i^{\mathcal{R}}}(r_i^{\mathcal{R}|x}) \text{ and } y \in C^{\mathcal{R}}\} \geq n\}$$

$$(\lesssim nr_i.C)^{\mathcal{R}} := \{x \in \Delta^{\mathcal{R}} \mid \#\{y \in \Delta^{\mathcal{R}} \mid (x,y) \in \min_{\prec_i^{\mathcal{R}}}(r_i^{\mathcal{R}|x}) \text{ and } y \in C^{\mathcal{R}}\} \leq n\}$$

Hence, $\approx nr.C$ is just an abbreviation for $(\gtrsim nr.C) \sqcap (\lesssim nr.C)$.

With these new constructs, one can revisit the example above and define the concept $\lesssim 2\mathsf{hasSibling.Female}$, which is coherent in the given scenario.

Just as with $\forall$, $\gtrsim n$ and $\lesssim n$ induce ternary functions $f_{\gtrsim n} : \langle r_i^{\mathcal{R}}, C^{\mathcal{R}}, \prec_{r_i}^{\mathcal{R}} \rangle \mapsto (\gtrsim nr_i.C)^{\mathcal{R}}$ and $f_{\lesssim n} : \langle r_i^{\mathcal{R}}, C^{\mathcal{R}}, \prec_{r_i}^{\mathcal{R}} \rangle \mapsto (\lesssim nr_i.C)^{\mathcal{R}}$. We then have:

Proposition 2. *$f_{\gtrsim n}$ is monotone in its first two arguments (the participating role and concept extensions) and non-monotonic in its third argument (the participating preference order). $f_{\lesssim n}$ is non-monotonic in its first two arguments and monotone in its third argument.*

Theorem 3. *$\mathcal{ALCQ}^{\gtrsim}$ has the finite-model property and is therefore decidable.*

Theorem 4. *In $\mathcal{ALCQ}^{\gtrsim}$, concept satisfiability and subsumption w.r.t. acyclic TBoxes are* PSPACE*-complete problems. Concept satisfiability and subsumption w.r.t. general TBoxes are* EXPTIME*-complete problems.*

4.3 Plausible Role Inclusion and Role Characteristics

Some expressive DLs [19] allow for the specification of (atomic) role inclusions of the form $r_i \sqsubseteq r_j$, whose semantics is given by $\mathcal{I} \Vdash r_i \sqsubseteq r_j$ if and only if $r_i^{\mathcal{I}} \subseteq r_j^{\mathcal{I}}$, capturing the intuition according to which an r_i-relationship is a special case of an r_j-one. $\mathcal{ALCHQ}$ denotes the extension of $\mathcal{ALCQ}$ with role hierarchies.

That this characterisation of role subsumption does not suffice for reasoning under uncertainty is already clear from the vast literature on non-monotonic reasoning. As a concrete example in a DL setting, consider the role inclusions $\mathsf{guardianOf} \sqsubseteq \mathsf{parentOf}$ and $\mathsf{parentOf} \sqsubseteq \mathsf{progenitorOf}$. In the absence of a construct to account for exceptions to these inclusions, it follows that $\mathsf{guardianOf} \sqsubseteq \mathsf{progenitorOf}$, a clearly undesirable consequence.

In order to cope with such cases, we here introduce plausible role inclusions of the form $r_i \sqsubsetsim r_j$, inspired by the meaning of defeasible consequence in propositional logic [32] and by defeasible concept subsumption in DLs [14], with the reading 'usually, a relationship via r_i is also an r_j-relationship.'

Let $\mathcal{ALC\tilde{H}Q}$ denote $\mathcal{ALCHQ}$ extended with plausible atomic role inclusions. Here, too, our r-ordered interpretations come in handy in providing an intuitive semantics for such a construct:

$$\mathcal{R} \Vdash r_i \sqsubsetapprox r_j \text{ if and only if } \min_{\prec_i^{\mathcal{R}}} r_i^{\mathcal{R}} \subseteq r_j^{\mathcal{R}}$$

With the notion of plausible role inclusion, stating $\mathsf{guardianOf} \sqsubsetapprox \mathsf{parentOf}$ and $\mathsf{parentOf} \sqsubsetapprox \mathsf{progenitorOf}$ captures in a better way the expected intuition in the above example.

Monotonicity for role inclusions coincides with transitivity: $r_i \sqsubseteq r_j$ and $r_j \sqsubseteq r_k$ implies $r_i \sqsubseteq r_k$. That is, strengthening r_j to r_i preserves the role subsumption by r_k. Monotonicity of plausible role inclusions can be defined analogously, i.e., if $r_i \sqsubseteq r_j$ and $r_j \sqsubsetapprox r_k$, then $r_i \sqsubsetapprox r_k$. It then follows that $\sqsubsetapprox$, as expected, fails the monotonicity property:

Proposition 3. *Plausible atomic role inclusion in $\mathcal{ALC\tilde{H}Q}$ is non-monotonic.*

Theorem 5. *$\mathcal{ALC\tilde{H}Q}$ has the finite-model property and is therefore decidable.*

Theorem 6. *In $\mathcal{ALC\tilde{H}Q}$, concept satisfiability and subsumption w.r.t. general TBoxes are* EXPTIME*-complete problems.*

Besides role hierarchies, some DLs also allow for the expression of *role characteristics* such as functionality, transitivity, disjointness, and others, often via the special notation $\mathsf{func}(r_i)$, $\mathsf{trans}(r_i)$, $\mathsf{disj}(r_i, r_j)$, etc, of which the intuition is that "r_i is functional", "r_i is transitive", "r_i and r_j are disjoint", and so on. Semantically, this corresponds to requiring, in every interpretation $\mathcal{I}$, that $r_i^{\mathcal{I}}$ be a function, that $r_i^{\mathcal{I}}$ be a transitive relation, that $r_i^{\mathcal{I}} \cap r_j^{\mathcal{I}} = \emptyset$, etc.

It turns out that, in real-world applications, such general, universal requirements can be too strong, as we have seen in the Introduction for the roles marriedTo (functional) and partOf (transitive). In each of these cases, the property under consideration does not hold globally, but it is still interesting to be able to express that it usually holds, or that it holds at least for the typical instances of the relation. We can achieve that in our framework via defeasible versions of the above characteristic specifiers, namely $\widetilde{\mathsf{func}}(r_i)$, $\widetilde{\mathsf{trans}}(r_i)$ and $\widetilde{\mathsf{disj}}(r_i, r_j)$, of which the intuition is that, respectively, "r_i is normally functional", "r_i is normally transitive" and "r_i and r_j are normally disjoint". The semantics of such constructs could be taken as: $\min_{\prec_i^{\mathcal{R}}} r_i^{\mathcal{R}}$ is functional, $\min_{\prec_i^{\mathcal{R}}} r_i^{\mathcal{R}}$ is transitive, $\min_{\prec_i^{\mathcal{R}}} r_i^{\mathcal{R}} \cap \min_{\prec_j^{\mathcal{R}}} r_j^{\mathcal{R}} = \emptyset$.

Theorem 7. *$\mathcal{ALCHQ}$ with defeasible role characteristics has the finite-model property and is therefore decidable.*

Theorem 8. *In $\mathcal{ALCHQ}$ with defeasible role characteristics, concept satisfiability and subsumption w.r.t. general TBoxes are* EXPTIME*-complete problems.*

As in the classical case, it turns out that role-characteristics axioms are just syntactic sugar, since all role properties can be expressed using the constructors we have previously introduced. For instance, that a role r is usually functional can be captured via plausible qualified number restrictions (see Section 4.2) by stating axioms of the form $\top \sqsubseteq \lesssim 1r.\top$ in the TBox.

An alternative characterisation of defeasible transitivity can be obtained in terms of role composition and defeasible role subsumption. This, of course, requires a generalisation of preferences on role names to *operations* on roles so that one can talk about e.g. the most preferred pairs of a compound relation. This is what we address in the remainder of the present section.

Given an r-ordered interpretation $\mathcal{R}$ and role names r_1 and r_2, together with their respective preference relations $\prec^{\mathcal{R}}_{r_1}$ and $\prec^{\mathcal{R}}_{r_2}$, the following are questions that naturally arise in the context of role composition: What is $\prec^{\mathcal{R}}_{r_1 \circ r_2}$? Can $\prec^{\mathcal{R}}_{r_1 \circ r_2}$ be defined in terms of $\prec^{\mathcal{R}}_{r_1}$ and $\prec^{\mathcal{R}}_{r_2}$? More generally, do $\prec^{\mathcal{R}}_{r_1}, \ldots, \prec^{\mathcal{R}}_{r_n}$ completely define the respective preference relation associated with any composition of $r_1, \ldots, r_n$?

Intuitively, a tuple is more plausible in a composed relation if it arises as the composition of two more preferred tuples in the component relations, and it does not also arise as the composition of two less preferred tuples. The latter condition is necessary to eliminate conflicting preferences in the composite order. Technically, it ensures that the resulting relation is a strict partial order. Formally,

$$\begin{aligned}\prec^{\mathcal{R}}_{r_1 \circ r_2} := \{(x_1 y_1, x_2 y_2) \mid & \text{ for some } z_1, z_2\ [(x_1 z_1, x_2 z_2) \in \prec^{\mathcal{R}}_{r_1} \text{ and } (z_1 y_1, z_2 y_2) \in \prec^{\mathcal{R}}_{r_2}] \\ & \text{and for no } z_1, z_2\ [(x_2 z_2, x_1 z_1) \in \prec^{\mathcal{R}}_{r_1} \text{ and } (z_2 y_2, z_1 y_1) \in \prec^{\mathcal{R}}_{r_2}]\}.\end{aligned}$$

As an example, a typical tuple in the relation $(\mathsf{hasChild} \circ \mathsf{hasChild})^{\mathcal{R}}$ could be a grandparent and biological grandchild.

Armed with a definition of plausible role composition, we can now provide an alternative characterisation of defeasible role transitivity: r_i is plausibly transitive if and only if $r_i \circ r_i \,\underset{\sim}{\sqsubset}\, r_i$. This definition requires only the most typical tuples in the composite relation $(r_i \circ r_i)^{\mathcal{R}}$ to be in $r_i^{\mathcal{R}}$, and is therefore not equivalent to the requirement that $\min_{\prec^{\mathcal{R}}_i} r_i^{\mathcal{R}}$ be transitive. Which of these two definitions is correct depends on what we want to model, and warrants further investigation.

4.4 Typicality of Roles

Plausible role inclusions of the form $r_i \,\underset{\sim}{\sqsubset}\, r_j$ (or, more generally, $r_1 \circ \cdots \circ r_k \,\underset{\sim}{\sqsubset}\, r_j$) carry an implicit notion of *typicality*, namely that typical r_is are r_js (or that the typical instances of $r_1 \circ \cdots \circ r_k$ are in the extension of r_j). Such a notion is implicit inasmuch as one cannot directly refer to the typical instances of r_i (or even of $r_1 \circ \cdots \circ r_k$) in the *object* language. (An analogous observation can be made about plausible concept inclusions of the form $C \,\underset{\sim}{\sqsubset}\, D$ — see Section 5.)

As has been argued in a propositional setting [9, 10], having an explicit notion of typicality at one's disposal comes in handy from a modeling perspective, besides increasing the expressive power of the language at no extra computational cost. The modeling interest translates into the freedom to refer to typicality anywhere within a sentence and not just in the antecedent (LHS) of 'implication-like' statements [16, 17], as with plausible subsumptions.

In a DL setting, this need is mainly felt when stating ABox assertions, namely in specifying that an individual is a typical instance of a class or that a pair of individuals is a typical instance of a role.

This issue has partially been addressed in the literature in that explicit notions of *concept* typicality have been introduced [5, 25], where with $\mathbf{T}(C)$ or $\mathbf{N}(C)$ one can refer, in both the TBox and the ABox, to the most typical (or most normal) members of a class. To the best of our knowledge, typicality of roles has never been considered before. Therefore, here we make a case for introducing a typicality operator for roles, with which one can capture the most normal or typical instances of a *relationship*.

Let $\star$ denote a unary operator on roles of which the intuition is precisely as motivated above and whose semantics is given by:

$$(\star r_i)^{\mathcal{R}} := \min_{\prec_i^{\mathcal{R}}} r_i^{\mathcal{R}}$$

In a logic equipped with $\star$, plausible role subsumption becomes redundant, since for every r_i, r_j, $\mathcal{R} \Vdash r_i \sqsubsetsim r_j$ iff $\mathcal{R} \Vdash \star r_i \sqsubseteq r_j$. A concrete example is $\star\mathsf{parentOf} \sqsubseteq \mathsf{progenitorOf}$, which we have seen in the previous section. Other examples involving the use of $\star$ are $\star\mathsf{marriedTo} \sqsubseteq \star\mathsf{hasPartner}$ (with typicality also in the RHS) and $\star\mathsf{marriedTo}(\mathsf{john}, \mathsf{mary})$ (an explicit instantiation of the typical portion of a role).

Let $f_\star : \langle r_i^{\mathcal{R}}, \prec_i^{\mathcal{R}} \rangle \mapsto min_{\prec_i^{\mathcal{R}}} r_i^{\mathcal{R}}$ denote the function induced by $\star$.

Proposition 4. *$f_\star$ is monotone (increasing) in its first argument, monotone (decreasing) in its second argument, and non-monotonic in general.*

Theorem 9. *$\mathcal{ALCHQ}$ with role typicality has the finite-model property.*

Theorem 10. *In $\mathcal{ALCHQ}$ with role typicality, concept satisfiability and subsumption w.r.t. general TBoxes are* EXPTIME*-complete problems.*

We conclude this section with a remark on further fruitfulness of role typicality from a modeling perspective. First, in DLs also allowing for Boolean operators on roles, with a statement of the form $\star r_i \sqcap \star r_j \sqsubseteq \bot$ one can express plausible role disjointness (see Section 4.3). Second, role typicality may be useful in further constraining certain roles via role constructors, e.g. $\star\mathsf{hasGrandChild} \sqsubseteq \star\mathsf{hasChild} \circ \star\mathsf{hasChild}$ (typical grandparenthoods are compositions of typical parenthoods). Both cases go beyond $\mathcal{ALCHQ}$ and we shall leave for future work.

5 Embedding Plausible Concept Subsumption

As an approach to the formalisation of defeasible inheritance in DLs, Britz et al. [14] introduced the notion of *plausible concept subsumption*, which is captured by statements of the form $C \sqsubsetsim D$, read "usually, C is subsumed by D". Building up on the work by Kraus et al. [32] in the propositional case, Britz et al. [12] have put forward the following list of properties that $\sqsubsetsim$ ought to satisfy in order to be considered as appropriate in a non-monotonic setting:

$$\text{(Cons)}\ \top \not\sqsubsetsim \bot \quad \text{(Ref)}\ C \sqsubsetsim C \quad \text{(LLE)}\ \frac{\models C \equiv D,\ C \sqsubsetsim E}{D \sqsubsetsim E} \quad \text{(And)}\ \frac{C \sqsubsetsim D,\ C \sqsubsetsim E}{C \sqsubsetsim D \sqcap E}$$

$$\text{(Or)}\ \frac{C \precsim E,\ D \precsim E}{C \sqcup D \precsim E} \quad \text{(RW)}\ \frac{C \precsim D,\ \models D \sqsubseteq E}{C \precsim E} \quad \text{(CM)}\ \frac{C \precsim D,\ C \precsim E}{C \sqcap D \precsim E}$$

The last six properties are the obvious translations of the properties for preferential consequence relations proposed by Kraus et al. [32] in the propositional setting. They have been discussed at length in the literature for both the propositional and the DL cases [26, 32, 33] and we shall not do so here.

A plausible concept subsumption $\precsim$ satisfying all seven properties above is called a *preferential* subsumption. One can require $\precsim$ to satisfy other properties as well. Of particular interest is the property of rational monotonicity below:

$$\text{(RM)}\ \frac{C \precsim D,\ C \not\precsim \neg C'}{C \sqcap C' \precsim D}$$

A plausible subsumption also satisfying (RM) is called a *rational subsumption*.

The intuition for the semantics of a statement of the form $C \precsim D$ is that those most typical C-objects are also D-objects. In Britz et al.'s approach, this is captured by placing a preference relation on the domain $\Delta^{\mathcal{I}}$ of every DL interpretation and evaluating $C \precsim D$ to true whenever the minimal C-objects are included in $D^{\mathcal{I}}$.

In what follows, we show one possible way in which plausible concept inclusions can be given a semantics within our r-ordered interpretations framework.

The starting point is to also allow for a *universal role* u and *role identity* constructs of the form *id*(C) [19], where $C \in \mathcal{L}_{\mathcal{ALC}}$, and of which the semantics is given by

$$u^{\mathcal{R}} := \Delta^{\mathcal{R}} \times \Delta^{\mathcal{R}} \qquad id(C)^{\mathcal{R}} := \{(x, x) \in \Delta^{\mathcal{R}} \times \Delta^{\mathcal{R}} \mid x \in C^{\mathcal{R}}\}$$

Next, one has to place an ordering $\prec_u^{\mathcal{R}}$ on the elements of $u^{\mathcal{R}}$ in the same way as for the other role interpretations. The intuition of doing so is that the most normal *id*(C)-pairs w.r.t. $\prec_u^{\mathcal{R}}$ correspond (implicitly) to the most normal C-objects, i.e., we get an ordering on the elements of $C^{\mathcal{R}}$ induced by the absolute ordering on the elements of $u^{\mathcal{R}}$. Armed with these ideas, we can provide a semantics for the notion of plausible concept inclusion as follows:

$$\mathcal{R} \Vdash C \precsim D \text{ if and only if } \min_{\prec_u^{\mathcal{R}}} id(C)^{\mathcal{R}} \subseteq id(D)^{\mathcal{R}}$$

Proposition 5. $\precsim$ *is strictly ampliative and non-monotonic.*

Proposition 6. $\precsim$ *is a preferential subsumption relation.*

If we also require $\prec_u^{\mathcal{R}}$ to be a *modular* order, then the above construction delivers a rational $\precsim$. Previous results for Rational Closure in DLs [12] carry over to $\mathcal{ALC}$ with plausible concept inclusions as defined above.

Theorem 11. *$\mathcal{ALCQ}$ extended with plausible concept inclusions has the finite-model property and is therefore decidable.*

Theorem 12. *In $\mathcal{ALCQ}$ with plausible concept inclusions, concept satisfiability and subsumption w.r.t. general TBoxes are* EXPTIME*-complete problems.*

6 Related and Future Work

We start by observing that the operators we have introduced here do not aim at providing a formal account of the notion of *most*, as addressed in the study of generalised quantifiers [36] and, more recently, in a modal context by Veloso et al. [44] and Askounis et al. [1]. Clearly, our defeasible operators are not about degrees of truth as has been studied in fuzzy logics, nor about degrees of possibility and necessity as addressed by possibilistic logics [24]. They rather relate to and generalise the notions of defeasible modalities [17, 18] and defeasible quantifiers [13] we studied previously.

In a sense, the notions we investigated here can be seen as the qualitative counterpart of possibilistic modalities [34, 35]. There, each possible world w is associated with a *possibility distribution* $\pi_w : W \longrightarrow [0, 1]$, the intuition of which is to capture the degree of likelihood (in terms of belief) of all possible worlds w.r.t. w. In that setting, the pairs (w, w') for which $\pi_w(w')$ is maximal correspond here to the most preferred pairs in the interpretation of a *single* role name.

In this paper, we have assumed $\mathcal{ALC}$, $\mathcal{ALCQ}$ or $\mathcal{ALCHQ}$ as the underlying DL and we have investigated individual extensions of each one with the constructors we introduced. As a next step, we shall consider different combinations of our defeasible constructs, also together with other DL operators not considered here, like inverse roles, and study the resulting computational properties.

Finally, here we have not addressed the question as to what an appropriate notion of non-monotonic entailment for the different extensions of $\mathcal{ALC}$ with defeasible operators is, especially in the presence of ABoxes. Indeed, in this paper we have contented ourselves with the standard (Tarskian) definition, which is monotonic (and therefore not suitable in some contexts). The recent extensions of the notion of Rational Closure [33] by Booth et al. [8, 9], Casini et al. [21] and Giordano et al. [27, 28] may provide us with a springboard with which to investigate this matter in the more expressive languages we introduced here.

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