

First Steps in \mathcal{EL} Contraction

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Abstract

In this work we make a preliminary investigation of belief contraction in the \mathcal{EL} family of description logics. Based on previous work on propositional Horn logic contraction, here we state the main definitions for contraction in a description logic extending it, and point out the main issues involved when addressing belief set contraction in \mathcal{EL} .

1 Motivation

Belief change is a subarea of knowledge representation concerned with describing how an intelligent agent ought to change its beliefs about the world in the face of new and possibly conflicting information. Arguably the most influential work in this area is the so-called AGM approach [Alchourrón *et al.*, 1985; Gärdenfors, 1988] which focuses on two types of belief change: *revision*, in which an agent has to keep its set of beliefs consistent while incorporating new information into it, and *contraction*, in which an agent has to give up some of its beliefs in order to avoid drawing unwanted conclusions.

Our primary reason for focusing on this topic is because of its application to constructing and repairing ontologies in description logics (DLs) [Baader *et al.*, 2003]. A typical scenario involves an ontology engineer teaming up with an expert to construct an ontology related to the domain of expertise of the latter with the aid of an ontology engineering tool such as SWOOP [<http://code.google.com/p/swoop>] or Protégé [<http://protege.stanford.edu>]. An example of this is the medical ontology SNOMED [Spackman *et al.*, 1997], whose formal substratum is the less expressive description logic \mathcal{EL} [Baader, 2003].

One of the principal methods for testing the quality of a constructed ontology is for the domain expert to inspect and verify the computed *subsumption hierarchy*. Correcting such errors involves the expert pointing out that certain subsumptions are missing from the subsumption hierarchy, while others currently occurring in the subsumption hierarchy ought not to be there. A concrete example of this involves SNOMED, which classifies the concept *Amputation-of-Finger* as being subsumed by the concept *Amputation-of-Arm*. Finding a solution to problems such as these is known as *repair* in the DL commu-

nity [Schlobach and Cornet, 2003], but it can also be seen as the problem of *contracting* by the subsumption statement *Amputation-of-Finger* \sqsubseteq *Amputation-of-Arm*.

The scenario also illustrates why we are concerned with belief contraction of belief *sets* (logically closed theories) and not *belief base* contraction [Hansson, 1999]. In practice, ontologies are not constructed by writing DL axioms, but rather using ontology editing tools, from which the axioms are generated automatically. Because of this, from the knowledge engineer's perspective, it is the belief set and not the axioms from which the theory is generated that is important.

2 Classical Belief Contraction

Before establishing our constructions, we give a very brief summary of classical belief contraction. For that we assume the reader is familiar with Classical Propositional Logic. Given a finitely generated propositional language $\mathcal{L}_{\mathcal{P}}$, if $X \subseteq \mathcal{L}_{\mathcal{P}}$, the set of sentences logically entailed by X is denoted by $Cn(X)$. A *belief set* is a logically closed set, i.e., for a belief set X , $X = Cn(X)$.

AGM [Alchourrón *et al.*, 1985] is the best-known approach to contraction. It gives a set of postulates characterizing all rational contraction functions. The aim is to describe belief contraction on the *knowledge level* of how beliefs are represented. Belief states are modelled by *belief sets* in a logic with a Tarskian consequence relation including classical propositional logic. The *expansion* of K by φ , $K + \varphi$, is defined as $Cn(K \cup \{\varphi\})$. *Contraction* is intended to represent situations in which an agent has to give up information from its current beliefs. Formally, belief contraction is a (partial) function from $\mathcal{P}(\mathcal{L}_{\mathcal{P}}) \times \mathcal{L}_{\mathcal{P}}$ to $\mathcal{P}(\mathcal{L}_{\mathcal{P}})$: the contraction of a belief set by a sentence yields a new set.

The AGM approach to contraction requires that the following set of postulates characterize *basic* contraction.

- (K-1) $K - \varphi = Cn(K - \varphi)$
- (K-2) $K - \varphi \subseteq K$
- (K-3) If $\varphi \notin K$, then $K - \varphi = K$
- (K-4) If $\not\models \varphi$, then $\varphi \notin K - \varphi$
- (K-5) If $\varphi \equiv \psi$, then $K - \varphi = K - \psi$
- (K-6) If $\varphi \in K$, then $(K - \varphi) + \varphi = K$

Various methods exist for constructing basic AGM contraction. In this paper we focus on the use of *remainder sets*.

Definition 2.1 For a belief set K , $X \in K \downarrow \varphi$ iff (i) $X \subseteq K$, (ii) $X \not\models \varphi$, and (iii) for every X' s.t. $X \subset X' \subseteq K$, $X' \models \varphi$. We call the elements of $K \downarrow \varphi$ remainder sets of K w.r.t. φ .

Remainder sets are belief sets, and $K \downarrow \varphi = \emptyset$ iff $\models \varphi$. AGM presupposes the existence of a suitable selection function for choosing between possibly different remainder sets.

Definition 2.2 A selection function σ is a function from $\mathcal{P}(\mathcal{P}(\mathcal{L}_P))$ to $\mathcal{P}(\mathcal{P}(\mathcal{L}_P))$ such that $\sigma(K \downarrow \varphi) = \{K\}$, if $K \downarrow \varphi = \emptyset$, and $\emptyset \neq \sigma(K \downarrow \varphi) \subseteq K \downarrow \varphi$ otherwise.

Selection functions provide a mechanism for identifying the remainder sets judged to be most appropriate, and the resulting contraction is then obtained by taking the intersection of the chosen remainder sets.

Definition 2.3 For σ a selection function, $-\sigma$ is a partial meet contraction iff $K -\sigma \varphi = \bigcap \sigma(K \downarrow \varphi)$.

One of the fundamental results of AGM contraction is a representation theorem which shows that partial meet contraction corresponds exactly with the six basic AGM postulates.

Theorem 2.1 ([Gärdenfors, 1988]) Every partial meet contraction satisfies (K-1)–(K-6). Conversely, every contraction function satisfying (K-1)–(K-6) is a partial meet contraction.

Two subclasses of partial meet deserve special mention.

Definition 2.4 Given a selection function σ , $-\sigma$ is a maxichoice contraction iff $\sigma(K \downarrow \varphi)$ is a singleton set. It is a full meet contraction iff $\sigma(K \downarrow \varphi) = K \downarrow \varphi$ whenever $K \downarrow \varphi \neq \emptyset$.

Clearly full meet contraction is unique, while maxichoice contraction usually is not.

3 The Description Logic \mathcal{EL}

The language of \mathcal{EL} ($\mathcal{L}_{\mathcal{EL}}$) is a sublanguage of the description logic \mathcal{ALC} [Baader *et al.*, 2003] built upon a set of atomic *concept names* $\{A, B, \dots\}$ and a set of *role names* $\{R, S, \dots\}$, together with the constructors \sqcap and \exists . Among the concept names are the distinguished concepts \top and \perp . Complex concepts are denoted by C, D, \dots and are constructed according to the rule

$$C ::= A \mid \top \mid \perp \mid C \sqcap C \mid \exists R.C$$

The semantics of \mathcal{EL} is the standard set theoretic Tarskian semantics. An *interpretation* is a structure $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, where $\Delta^{\mathcal{I}}$ is a non-empty set called the *domain*, and $\cdot^{\mathcal{I}}$ is an *interpretation function* mapping concept names A to subsets $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$ and mapping role names R to binary relations $R^{\mathcal{I}}$ over $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$:

$$\begin{aligned} A^{\mathcal{I}} &\subseteq \Delta^{\mathcal{I}}, \quad R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}, \\ \top^{\mathcal{I}} &= \Delta^{\mathcal{I}}, \quad \perp^{\mathcal{I}} = \emptyset \end{aligned}$$

Given an interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, $\cdot^{\mathcal{I}}$ is extended to interpret complex concepts in the following way:

$$\begin{aligned} (C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}}, \\ (\exists R.C)^{\mathcal{I}} &= \{a \in \Delta^{\mathcal{I}} \mid \exists b.(a, b) \in R^{\mathcal{I}} \text{ and } b \in C^{\mathcal{I}}\} \end{aligned}$$

Given \mathcal{EL} concepts C, D , $C \sqsubseteq D$ is a *general concept inclusion axiom* (GCI for short). $C \equiv D$ is an abbreviation for both $C \sqsubseteq D$ and $D \sqsubseteq C$. An \mathcal{EL} -TBox \mathcal{T} is a set of GCIs.

An interpretation \mathcal{I} satisfies an axiom $C \sqsubseteq D$ (noted $\mathcal{I} \models C \sqsubseteq D$) iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$. $\mathcal{I} \models C \equiv D$ iff $C^{\mathcal{I}} = D^{\mathcal{I}}$.

An interpretation \mathcal{I} is a *model* of a TBox \mathcal{T} (noted $\mathcal{I} \models \mathcal{T}$) iff \mathcal{I} satisfies every axiom in \mathcal{T} . An axiom $C \sqsubseteq D$ is a *consequence* of a TBox \mathcal{T} , noted $\mathcal{T} \models C \sqsubseteq D$, iff for every interpretation \mathcal{I} , if $\mathcal{I} \models \mathcal{T}$, then $\mathcal{I} \models C \sqsubseteq D$. Given a TBox \mathcal{T} , $Cn(\mathcal{T}) = \{C \sqsubseteq D \mid \mathcal{T} \models C \sqsubseteq D\}$. An \mathcal{EL} belief set is then a TBox $\mathcal{T} = Cn(\mathcal{T})$.

For more details on \mathcal{EL} , the reader is referred to the work by Baader [2003].

4 \mathcal{EL} Contraction

In this section we give the basic definitions for contraction in \mathcal{EL} . The presentation follows that for Horn contraction by Delgrande [2008] and Booth *et al.* [2009].

We want to be able to contract an \mathcal{EL} TBox \mathcal{T} with a set of GCIs Φ . The goal of contracting \mathcal{T} with Φ is the removal of at least one of the axioms in Φ . For full propositional logic, contraction with a set of sentences is not particularly interesting since it amounts to contracting by their conjunction. For \mathcal{EL} it is interesting though, since the conjunction of the axioms in Φ is not always expressible as a single \mathcal{EL} axiom. Our starting point for defining contraction is in terms of remainder sets.

Definition 4.1 (\mathcal{EL} Remainder Sets) For a belief set \mathcal{T} , $X \in \mathcal{T} \downarrow_{\mathcal{EL}} \Phi$ iff (i) $X \subseteq \mathcal{T}$, (ii) $X \not\models \Phi$, and (iii) for every X' s.t. $X \subset X' \subseteq \mathcal{T}$, $X' \models \Phi$. We refer to the elements of $\mathcal{T} \downarrow_{\mathcal{EL}} \Phi$ as the \mathcal{EL} remainder sets of \mathcal{T} w.r.t. Φ .

\mathcal{EL} remainder sets exist since \mathcal{EL} is compact [Sofronie-Stokkermans, 2006] and has a Tarskian consequence relation. It is easy to verify that all \mathcal{EL} remainder sets are belief sets. Also, $\mathcal{T} \downarrow_{\mathcal{EL}} \Phi = \emptyset$ iff $\models \Phi$.

We now proceed to define selection functions to be used for \mathcal{EL} partial meet contraction.

Definition 4.2 (\mathcal{EL} Selection Functions) A partial meet \mathcal{EL} selection function σ is a function from $\mathcal{P}(\mathcal{P}(\mathcal{L}_{\mathcal{EL}}))$ to $\mathcal{P}(\mathcal{P}(\mathcal{L}_{\mathcal{EL}}))$ s.t. $\sigma(\mathcal{T} \downarrow_{\mathcal{EL}} \Phi) = \{\mathcal{T}\}$ if $\mathcal{T} \downarrow_{\mathcal{EL}} \Phi = \emptyset$, and $\emptyset \neq \sigma(\mathcal{T} \downarrow_{\mathcal{EL}} \Phi) \subseteq \mathcal{T} \downarrow_{\mathcal{EL}} \Phi$ otherwise.

Using these selection functions, we define partial meet \mathcal{EL} contraction.

Definition 4.3 (Partial Meet \mathcal{EL} Contraction) Given a partial meet \mathcal{EL} selection function σ , $-\sigma$ is a partial meet \mathcal{EL} contraction iff $\mathcal{T} -\sigma \Phi = \bigcap \sigma(\mathcal{T} \downarrow_{\mathcal{EL}} \Phi)$.

We also consider two special cases.

Definition 4.4 (\mathcal{EL} Maxichoice and Full Meet) Given a partial meet \mathcal{EL} selection function σ , $-\sigma$ is a maxichoice \mathcal{EL} contraction iff $\sigma(\mathcal{T} \downarrow_{\mathcal{EL}} \Phi)$ is a singleton set. It is a full meet \mathcal{EL} contraction iff $\sigma(\mathcal{T} \downarrow_{\mathcal{EL}} \Phi) = \mathcal{T} \downarrow_{\mathcal{EL}} \Phi$ when $\mathcal{T} \downarrow_{\mathcal{EL}} \Phi \neq \emptyset$.

Example 4.1 Let $\mathcal{T} = Cn(\{A \sqsubseteq B, B \sqsubseteq \exists R.A\})$. Then $\mathcal{T} \downarrow_{\mathcal{EL}} \{A \sqsubseteq \exists R.A\} = \{\mathcal{T}'_1, \mathcal{T}'_2\}$, where $\mathcal{T}'_1 = Cn(\{A \sqsubseteq B\})$, and $\mathcal{T}'_2 = Cn(\{B \sqsubseteq \exists R.A, A \sqcap \exists R.A \sqsubseteq B\})$. So contracting with $\{A \sqsubseteq \exists R.A\}$ yields either \mathcal{T}'_1 , \mathcal{T}'_2 , or $\mathcal{T}'_1 \cap \mathcal{T}'_2 = Cn(\{A \sqcap \exists R.A \sqsubseteq B\})$.

As pointed out by Booth *et al.* [2009], maxichoice contraction is not enough for logics that are less expressive than full propositional logic, like e.g. Horn logic. The same argument holds here for \mathcal{EL} , as the following example shows.

Example 4.2 Recalling Example 4.1, when contracting $\{A \sqsubseteq \exists R.A\}$ from $\mathcal{T} = \text{Cn}(\{A \sqsubseteq B, B \sqsubseteq \exists R.A\})$, full meet yields $\mathcal{T}_{fm} = \text{Cn}(\{A \sqcap \exists R.A \sqsubseteq B\})$, while maxichoice yields either $\mathcal{T}_{mc}' = \text{Cn}(\{A \sqsubseteq B\})$ or $\mathcal{T}_{mc}'' = \text{Cn}(\{B \sqsubseteq \exists R.A, A \sqcap \exists R.A \sqsubseteq B\})$. Now consider the belief set $\mathcal{T}_i = \text{Cn}(\{A \sqcap \exists R.A \sqsubseteq B, A \sqcap B \sqsubseteq \exists R.A\})$. Clearly $\mathcal{T}_{fm} \subseteq \mathcal{T}_i \subseteq \mathcal{T}_{mc}''$, but no partial meet yields \mathcal{T}_i .

Our contention is that \mathcal{EL} contraction should include cases such as \mathcal{T}_i above. Given that full meet contraction is deemed to be appropriate, it stands to reason that any belief set \mathcal{T}_i bigger than it should also be seen as appropriate, *provided* that \mathcal{T}_i does not contain any irrelevant additions. But since \mathcal{T}_i is contained in some maxichoice contraction, \mathcal{T}_i cannot contain any irrelevant additions. After all, the maxichoice contraction contains only relevant additions, since it is an appropriate form of contraction. Hence \mathcal{T}_i is also an appropriate result of \mathcal{EL} contraction.

Definition 4.5 (Infra-Remainder Sets) For belief sets \mathcal{T} and X , $X \in \mathcal{T} \downarrow_{\mathcal{EL}} \Phi$ iff there is some $X' \in \mathcal{T} \downarrow_{\mathcal{EL}} \Phi$ s.t. $(\bigcap \mathcal{T} \downarrow_{\mathcal{EL}} \Phi) \subseteq X \subseteq X'$. We refer to the elements of $\mathcal{T} \downarrow_{\mathcal{EL}} \Phi$ as the infra-remainder sets of \mathcal{T} w.r.t. Φ .

Note that all remainder sets are also infra-remainder sets, and so is the intersection of any set of remainder sets. Indeed, the intersection of any set of infra-remainder sets is also an infra-remainder set. So the set of infra-remainder sets contains *all* belief sets between some remainder set and the intersection of all remainder sets.

Definition 4.6 (\mathcal{EL} Contraction) An infra-selection function τ is a function from $\mathcal{P}(\mathcal{P}(\mathcal{L}_{\mathcal{EL}}))$ to $\mathcal{P}(\mathcal{L}_{\mathcal{EL}})$ s.t. $\tau(\mathcal{T} \downarrow_{\mathcal{EL}} \Phi) = \mathcal{T}$ whenever $\models \Phi$, and $\tau(\mathcal{T} \downarrow_{\mathcal{EL}} \Phi) \in \mathcal{T} \downarrow_{\mathcal{EL}} \Phi$ otherwise. A contraction function $-\tau$ is an \mathcal{EL} contraction iff $\mathcal{T} -_{\tau} \Phi = \tau(\mathcal{T} \downarrow_{\mathcal{EL}} \Phi)$.

5 A Representation Result

Our representation result makes use of \mathcal{EL} versions of all of the basic AGM postulates, except for the Recovery Postulate ($K-6$). It is easy to see that \mathcal{EL} contraction does not satisfy Recovery. As an example, take $\mathcal{T} = \text{Cn}(\{A \sqsubseteq C\})$ and let $\Phi = \{A \sqcap B \sqsubseteq C\}$. Then $\mathcal{T} - \Phi = \text{Cn}(\emptyset)$ and so $(\mathcal{T} - \Phi) + \Phi = \text{Cn}(\{A \sqcap B \sqsubseteq C\}) \neq \mathcal{T}$. In place of Recovery we have a postulate that closely resembles Hansson's [1999] Relevance Postulate, and a postulate handling the case when trying to contract with a tautology.

$$(\mathcal{T}-1) \quad \mathcal{T} - \Phi = \text{Cn}(\mathcal{T} - \Phi)$$

$$(\mathcal{T}-2) \quad \mathcal{T} - \Phi \subseteq \mathcal{T}$$

$$(\mathcal{T}-3) \quad \text{If } \Phi \not\subseteq \mathcal{T}, \text{ then } \mathcal{T} - \Phi = \mathcal{T}$$

$$(\mathcal{T}-4) \quad \text{If } \not\models \Phi, \text{ then } \Phi \not\subseteq \mathcal{T} - \Phi$$

$$(\mathcal{T}-5) \quad \text{If } \text{Cn}(\Phi) = \text{Cn}(\Psi), \text{ then } \mathcal{T} - \Phi = \mathcal{T} - \Psi$$

$$(\mathcal{T}-6) \quad \text{If } \alpha \in \mathcal{T} \setminus (\mathcal{T} - \Phi), \text{ then there is a } \mathcal{T}' \text{ such that } \bigcap (\mathcal{T} \downarrow_{\mathcal{EL}} \Phi) \subseteq \mathcal{T}' \subseteq \mathcal{T}, \mathcal{T}' \not\models \Phi, \text{ and } \mathcal{T}' + \{\alpha\} \models \Phi$$

$$(\mathcal{T}-7) \quad \text{If } \models \Phi, \text{ then } \mathcal{T} - \Phi = \mathcal{T}$$

Postulates $(\mathcal{T}-1)$ – $(\mathcal{T}-5)$ are analogues of $(K-1)$ – $(K-5)$, while $(\mathcal{T}-6)$ states that all sentences removed from \mathcal{T} during a contraction of Φ must have been removed for a reason: adding them again brings back Φ . $(\mathcal{T}-7)$ simply states that contracting with a (possibly empty) set of tautologies leaves the initial belief set unchanged. We remark that $(\mathcal{T}-3)$ is actually redundant in the list, since it can be shown to follow mainly from $(\mathcal{T}-6)$.

Theorem 5.1 Every \mathcal{EL} contraction satisfies $(\mathcal{T}-1)$ – $(\mathcal{T}-7)$. Conversely, every contraction function satisfying $(\mathcal{T}-1)$ – $(\mathcal{T}-7)$ is an \mathcal{EL} contraction.

6 Concluding Remarks

Here we have laid the groundwork for contraction in the \mathcal{EL} family of description logics by providing a formal account of *basic* contraction of a TBox with a set of GCIs.

Here we focus only on *basic* contraction. For future work we plan to investigate \mathcal{EL} contraction for *full* AGM contraction, obtained by adding the *extended* postulates. We also plan to pursue our future investigations on repair in more expressive DLs.

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