

Pertinent Entailment

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We investigate a family of entailment relations that we call *pertinent entailments*. The pertinence between premiss and consequence proposed here is induced by an accessibility relation establishing a link (representing some form of pertinence) between premiss and consequence. We show that this notion can be elegantly captured using a simple modal logic without nested modalities.

Consider a propositional language and let $\mathcal{M} = \langle W, R \rangle$ be a frame, with W a set of worlds (truth assignments), and R a binary accessibility relation on W . Here \diamond denotes the normal modality for possibility [2], whereas \blacklozenge is the corresponding operator w.r.t. the *converse* accessibility relation. With $\models^{\mathcal{M}}$ we denote truth w.r.t. frame \mathcal{M} . Hence $X \models^{\mathcal{M}} Y$ reads “every X -world in \mathcal{M} is a Y -world”, and $X \models^{\mathcal{M}} \blacklozenge Y$ reads “from any X -world in \mathcal{M} one can reach some Y -world”. We define two *infra-classical* pertinent entailment relations as follows:

$$(1) X \overset{\frown}{\prec} Y \text{ iff } X \models^{\mathcal{M}} Y \text{ and } X \models^{\mathcal{M}} \blacklozenge Y \quad (2) X \overset{\blackleftarrow{\frown}}{\prec} Y \text{ iff } X \models^{\mathcal{M}} Y \text{ and } Y \models^{\mathcal{M}} \blacklozenge X$$

Intuitively, Definition (1) above says that premiss X is *forwardly pertinent* to the consequence Y if and only if X (classically) entails Y and whenever X is the case, there is a world accessible from that X -world in which Y holds. Definition (2) states that premiss X is *backwardly pertinent* to the consequence Y if and only if X entails Y (classically) and every Y -world is accessible from *some* X -world.

One road to infra-classicality has been studied extensively, that of *substructural* logics [3], which weaken the generating engine of axioms and inference rules for producing entailment pairs (X, Y) . Here we follow an alternative (not antagonistic) strategy: we first demand that X entails Y classically, but then (invoking R) more, trimming down the set of entailment pairs to infra-classicality.

Two well-known *bêtes noires* of relevance and relevant logicians [1] are the so-called *positive paradox*, $X \rightarrow (Y \rightarrow X)$, and *disjunctive syllogism*, $(\neg X \vee Y) \wedge X \models Y$. It turns out that our pertinent entailments $\overset{\frown}{\prec}$ and $\overset{\blackleftarrow{\frown}}{\prec}$ restrict these paradoxes in an interesting way. Moreover, $\overset{\frown}{\prec}$ and $\overset{\blackleftarrow{\frown}}{\prec}$ also possess other non-classical properties: (i) if $Y \equiv \top$, it is not the case that for every X we have $X \overset{\frown}{\prec} Y$ or $X \overset{\blackleftarrow{\frown}}{\prec} Y$, (ii) $\overset{\frown}{\prec}$ is *paraconsistent* in the following sense: if $X \overset{\frown}{\prec} Y$ and $X \equiv \perp$, then $Y \equiv \perp$. We also discuss the properties of these pertinent entailment relations w.r.t. inference rules traditionally considered in the literature.

From the standpoint of modal logic we investigate the properties that the accessibility relation R behind the definitions above must satisfy. For example, in the case of $\overset{\blackleftarrow{\frown}}{\prec}$ it turns out that R need not be reflexive, but to induce certain desirable properties, it has to be *serial* and *transitive*. This, together with the normality of the \diamond operator, gives us the modal logic KD4 [2]. Such a result suggests that our infra-classical pertinent entailments are employable in deontic and doxastic reasoning, while the notions of pertinence and belief/obligation overlap to some extent.

Keywords: Pertinence, non-classical consequence, modal logic, relevance and relevant logics

References

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