Next Steps in Reasoning Defeasibly over Description-Logic Ontologies

(based on joint work with R. Booth, K. Britz, G. Casini, T. Meyer, K. Moodley and U. Sattler)

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Defeasible reasoning over ontologies

$$\mathcal{T} = \left\{ \begin{array}{c} \mathsf{Parent} \equiv \exists \mathsf{hasChild}.\top, \\ \mathsf{Grandparent} \equiv \exists \mathsf{hasChild}.\mathsf{Parent}, \\ \mathsf{Husband} \sqsubseteq \exists \mathsf{marriedTo}.\mathsf{Woman}, \\ \mathsf{Wife} \sqsubseteq \exists \mathsf{marriedTo}.\mathsf{Man}, \\ \mathsf{Father} \equiv \mathsf{Man} \sqcap \mathsf{Parent}, \\ \mathsf{Mother} \equiv \mathsf{Woman} \sqcap \mathsf{Parent}, \\ \mathsf{Parent} \sqsubseteq \exists \mathsf{pays}.\mathsf{Nursery}, \\ \mathsf{SingleParent} \sqsubseteq \mathsf{Parent} \sqcap \forall \mathsf{hasPartner}.\bot, \\ \mathsf{SingleParent} \sqsubseteq \neg \exists \mathsf{pays}.\mathsf{Nursery}, \\ \mathsf{SingleParent} \sqcap \mathsf{Rich} \sqsubseteq \exists \mathsf{pays}.\mathsf{Nursery} \end{array} \right\} \qquad \mathcal{A} = \left\{ \begin{array}{c} \mathsf{Woman}(\mathsf{mary}), \quad \mathsf{Man}(\mathsf{john}), \\ \mathsf{marriedTo}(\mathsf{john}, \mathsf{mary}), \\ \mathsf{marriedTo}(\mathsf{mary}, \mathsf{john}), \\ \mathsf{hasChild}(\mathsf{mary}, \mathsf{alice}), \\ \mathsf{progenitorOf}(\mathsf{john}, \mathsf{alice}), \\ \mathsf{Woman}(\mathsf{jane}) \end{array} \right\}$$

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Problem

- $\mathcal{KB} \models \mathsf{SingleParent} \sqsubseteq \bot$
- In every interpretation $\mathcal{I}$, $\mathsf{SingleParent}^{\mathcal{I}} = \emptyset$
- I.e., there is no such thing as a single parent

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Big problem!

- $\mathcal{KB} \models \top \sqsubseteq \bot$
- $\mathcal{KB}$ is now inconsistent
- Collapse of reasoning

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Dealing explicitly with exceptions

- Does not scale well: the more exceptions, the less intuitive the default rule
- New explicit exceptions require remodeling
- Humans tend not to anticipate explicit exceptions

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Exceptionality and beyond

- Exceptions are pervasive in quotidian reasoning
- Classical reasoning does not cope with exceptions
- Worse: there is more to it than meets the eye!

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Ampliative and defeasible reasoning

- It is plausible to have $\mathcal{KB} \models \exists \mathsf{pays}.\mathsf{Nursery}(\mathsf{jane})$
- But upon learning SingleParent(jane), this conclusion should be dropped
- Actually, we would then want $\mathcal{KB} \models \neg\exists \mathsf{pays}.\mathsf{Nursery}(\mathsf{jane})$

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Ampliative and defeasible reasoning

- It is plausible to have $\mathcal{KB} \models \exists \mathsf{hasChild}.\exists \mathsf{pays}.\mathsf{Nursery}(\mathsf{john})$
- If we learn $\forall \mathsf{hasChild}.\mathsf{SingleParent}(\mathsf{john})$, this conclusion must be dropped
- Actually, we would then want $\mathcal{KB} \models \exists \mathsf{hasChild}.\neg \exists \mathsf{pays}.\mathsf{Nursery}(\mathsf{john})$

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The bottom line

- Need for endowing ontologies with non-monotonic reasoning capabilities
- Problem well studied in the propositional case (see '80s and '90s)
- Not so in more expressive logics: FOL, modal logic, DLs

Families of non-monotonic DLs

Default logic-based

- Baader & Hollunder (1993,1995), Padgham & Zhang (1993), Straccia (1993)

Circumscription-based

- Bonatti et al. (2009, 2011), Sengupta et al. (2011)

Priorities on axioms or rules

- Heymans & Vermeir (2002), Governatori (2004)

Logic programming-based

- Donini et al. (2003), Grosof et al. (2003), Knorr et al. (2012)

Possibility logic-based

- Qi et al. (2007,2013)

Preferential approaches

- Quantz et al. (1992), Giordano et al. (2007), Britz et al. (2008,2011), Casini & Straccia (2010), Pensel & Turhan (2017)

Outline

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Defeasible concept subsumption in $\mathcal{ALC}$

$$C \mathrel{\sqsubset\!\!\!\sim} D$$

Intuition

- Usually, C is subsumed by D (or typically, C is subsumed by D)
- Typical Cs are Ds (exceptional Cs need not)

Defeasible concept subsumption in $\mathcal{ALC}$

$$C \sqsubsetapprox D$$

Intuition

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- Typical Cs are Ds (exceptional Cs need not)

Example

- Parent $\sqsubsetapprox \exists$pays.Nursery
- SingleParent $\sqsubsetapprox \neg\exists$pays.Nursery
- SingleParent $\sqcap$ Rich $\sqsubsetapprox \exists$pays.Nursery

Defeasible concept subsumption in $\mathcal{ALC}$

$$C \sqsubsetsim D$$

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Example

- Parent $\sqsubsetsim$ $\exists$pays.Nursery
- SingleParent $\sqsubsetsim$ $\neg\exists$pays.Nursery
- SingleParent $\sqcap$ Rich $\sqsubsetsim$ $\exists$pays.Nursery

Expected behaviour

- To cope with exceptionality, $\sqsubsetsim$ should not be monotonic
- From $C \sqsubsetsim D$, one should not (in general) conclude $C \sqcap E \sqsubsetsim D$

Defeasible concept subsumption: semantics

Definition (Preferential interpretation)

A preferential interpretation is a tuple $\mathcal{P} := \langle \Delta^{\mathcal{P}}, \cdot^{\mathcal{P}}, \prec^{\mathcal{P}} \rangle$ where

- $\langle \Delta^{\mathcal{P}}, \cdot^{\mathcal{P}} \rangle$ is a DL interpretation
- $\prec^{\mathcal{P}}$ is a strict partial order on $\Delta^{\mathcal{P}}$
- for every $C \in \mathcal{L}$, if $C^{\mathcal{P}} \neq \emptyset$, then $\min_{\prec^{\mathcal{P}}}(C^{\mathcal{P}}) \neq \emptyset$ (i.e., $\prec^{\mathcal{P}}$ is smooth)

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Intuition

- Objects lower down in $\prec^{\mathcal{P}}$ are more normal than those higher up
- $\min_{\prec^{\mathcal{P}}}(C^{\mathcal{P}})$ denotes the most normal (or typical) elements in $C^{\mathcal{P}}$

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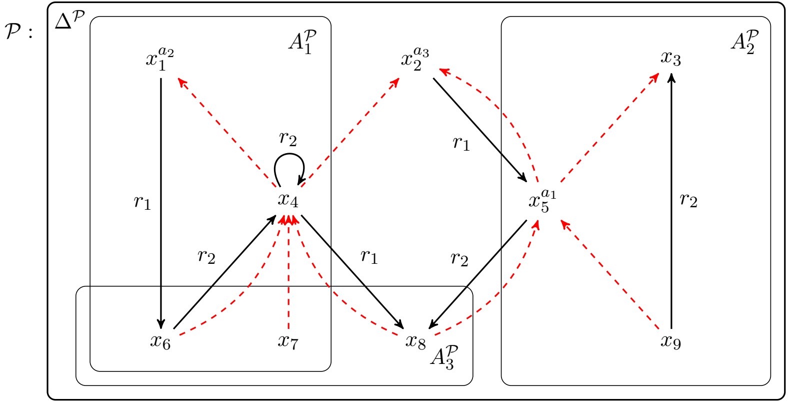
Definition

- $\mathcal{P} \Vdash C \sqsubseteq\!\!\!\sim D$ iff $\min_{\prec^{\mathcal{P}}}(C^{\mathcal{P}}) \subseteq D^{\mathcal{P}}$
- If $\mathcal{P} \Vdash \alpha$, then $\mathcal{P}$ is a model of α. $[\![\alpha]\!]$: set of all models of α.

Defeasible concept subsumption: semantics



- $\min_{\prec^{\mathcal{P}}}(A_1 \sqcup A_3)^{\mathcal{P}} = \{x_6, x_7, x_8\}$
- $\min_{\prec^{\mathcal{P}}}(\exists r_2.\top) = \{x_6, x_9\}$
- $\mathcal{P} \not\Vdash A_1 \sqsubseteq A_3$
- $\mathcal{P} \Vdash A_1 \mathrel{\sqsubseteq\!\!\!\sim} A_3$

Defeasible concept subsumption: properties

KLM-style properties (or ‘postulates’ or Gentzen-style rules)

(Cons) $\top \not\sqsubsetsim \bot$

(Ref) $C \sqsubsetsim C$

(LLE) $\dfrac{C \equiv D, \quad C \sqsubsetsim E}{D \sqsubsetsim E}$

(And) $\dfrac{C \sqsubsetsim D, \quad C \sqsubsetsim E}{C \sqsubsetsim D \sqcap E}$

(Or) $\dfrac{C \sqsubsetsim E, \quad D \sqsubsetsim E}{C \sqcup D \sqsubsetsim E}$

(RW) $\dfrac{C \sqsubsetsim D, \quad D \sqsubseteq E}{C \sqsubsetsim E}$

(CM) $\dfrac{C \sqsubsetsim D, \quad C \sqsubsetsim E}{C \sqcap D \sqsubsetsim E}$

(RM) $\dfrac{C \sqsubsetsim D, \quad C \not\sqsubsetsim \neg E}{C \sqcap E \sqsubsetsim D}$

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Theorem (Representation result)

'Soundness' and 'completeness' of the set of properties w.r.t. the preferential semantics

Defeasible ontologies

- TBox $\mathcal{T}$ enriched with defeasible concept subsumption statements

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Definition (Preferential entailment)

$\mathcal{KB} \models_P \alpha$ iff $[\![\mathcal{KB}]\!] \subseteq [\![\alpha]\!]$

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$$\mathcal{T} = \left\{ \begin{array}{c} \mathsf{Parent} \equiv \exists \mathsf{hasChild}.\top, \\ \mathsf{Grandparent} \equiv \exists \mathsf{hasChild}.\mathsf{Parent}, \\ \mathsf{Husband} \sqsubseteq \exists \mathsf{marriedTo}.\mathsf{Woman}, \\ \mathsf{Wife} \sqsubseteq \exists \mathsf{marriedTo}.\mathsf{Man}, \\ \mathsf{Father} \equiv \mathsf{Man} \sqcap \mathsf{Parent}, \\ \mathsf{Mother} \equiv \mathsf{Woman} \sqcap \mathsf{Parent}, \\ \mathsf{Parent} \mathrel{\sqsubseteq\!\!\!\sim} \exists \mathsf{pays}.\mathsf{Nursery}, \\ \mathsf{SingleParent} \sqsubseteq \mathsf{Parent}, \\ \mathsf{SingleParent} \mathrel{\sqsubseteq\!\!\!\sim} \neg \exists \mathsf{pays}.\mathsf{Nursery}, \\ \mathsf{SingleParent} \sqcap \mathsf{Rich} \mathrel{\sqsubseteq\!\!\!\sim} \exists \mathsf{pays}.\mathsf{Nursery} \end{array} \right\}, \mathcal{A} = \left\{ \begin{array}{c} \mathsf{Woman}(\mathsf{mary}), \\ \mathsf{Man}(\mathsf{john}), \\ \mathsf{marriedTo}(\mathsf{john}, \mathsf{mary}), \\ \mathsf{marriedTo}(\mathsf{mary}, \mathsf{john}), \\ \mathsf{hasChild}(\mathsf{mary}, \mathsf{alice}), \\ \mathsf{progenitorOf}(\mathsf{john}, \mathsf{alice}), \\ \mathsf{Woman}(\mathsf{jane}), \\ \mathsf{SingleParent}(\mathsf{alice}) \end{array} \right\}$$

$\not\models_P \mathsf{SingleParent} \sqsubseteq \bot$

$\not\models_P \top \sqsubseteq \bot$

$\models_P \mathsf{SingleParent} \sqcap \exists \mathsf{hasChild}.\top \mathrel{\sqsubseteq\!\!\!\sim} \neg \exists \mathsf{pays}.\mathsf{Nursery}$

$\models_P \mathsf{Mother}(\mathsf{mary})$

Definition (Preferential entailment)

$\mathcal{KB} \models_P \alpha$ iff $[\![\mathcal{KB}]\!] \subseteq [\![\alpha]\!]$

Defeasible ontologies

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$\not\models_P$ SingleParent $\sqsubseteq \bot$

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$\models_P$ SingleParent $\sqcap \exists$hasChild.$\top \precsim \neg\exists$pays.Nursery

$\models_P$ Mother(mary)

$\not\models_P$ Parent $\sqcap$ Rich $\precsim \exists$pays.Nursery

$\not\models_P \exists$pays.Nursery(mary)

Definition (Preferential entailment)

$\mathcal{KB} \models_P \alpha$ iff $[\![\mathcal{KB}]\!] \subseteq [\![\alpha]\!]$

Defeasible ontologies

Basic requirements for an appropriate notion of entailment

- Ampliativeness
- Defeasibility

Defeasible ontologies

Basic requirements for an appropriate notion of entailment

- Ampliativeness
- Defeasibility

Various ways to achieve this

- Vast literature on NMR

General pattern

- See some models of $\mathcal{KB}$ as ‘more important’

Defeasible ontologies

Basic requirements for an appropriate notion of entailment

- Ampliativeness
- Defeasibility

Various ways to achieve this

- Vast literature on NMR

General pattern

- See some models of $\mathcal{KB}$ as 'more important'

In our context

- To be important $=$ to maximise typicality (or normality)
- Idea: prefer models in which objects are as typical as possible

Defeasible ontologies

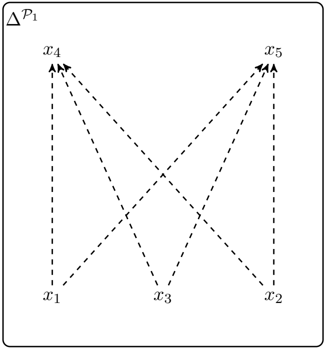
Preferring maximal typicality

Example

Let $\mathcal{P}_1 = \langle \Delta^{\mathcal{P}_1}, \cdot^{\mathcal{P}_1}, \prec^{\mathcal{P}_1} \rangle$ and $\mathcal{P}_2 = \langle \Delta^{\mathcal{P}_2}, \cdot^{\mathcal{P}_2}, \prec^{\mathcal{P}_2} \rangle$ be such that

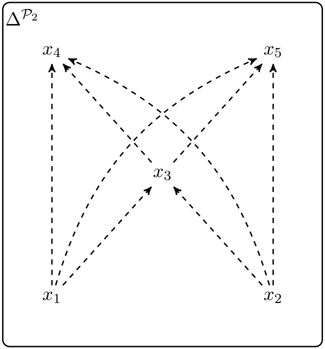
- $\Delta^{\mathcal{P}_1} = \Delta^{\mathcal{P}_2} = \{x_i \mid 1 \leq i \leq 5\}$ (same domain!), $\cdot^{\mathcal{P}_1} = \cdot^{\mathcal{P}_2}$, $\prec^{\mathcal{P}_1}$ and $\prec^{\mathcal{P}_2}$ as below

$\mathcal{P}_1$:



$\lhd$

$\mathcal{P}_2$:



Defeasible ontologies

Definition (Rational entailment)

$\mathcal{KB} \models_R \alpha$ iff $\min_{\lhd}[\![\mathcal{KB}]\!] \subseteq [\![\alpha]\!]$

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- Implementation: DIP (`github.com/kodymoodley/defeasibleinferenceplatform`)
- Experimental results: good performance scalability in practice

Limitations

Representation issues

- Defeasibility of argument forms, only: $C \mathrel{\sqsubset\!\!\!\sim} D$
- Single preference ordering on objects of the domain

Limitations

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Failure of properties

$$(\exists \mathsf{M})\ \frac{C \precsim D}{\exists r.C \precsim \exists r.D}$$

$$(\forall \mathsf{M})\ \frac{C \precsim D}{\forall r.C \precsim \forall r.D}$$

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Current work

- Strengthen the preference relation on models
- Prefer models of $\mathcal{KB}$ in which also $r^{\mathcal{P}}(x)$ are as typical as possible
- New representation theorems

Outline

A broader take on exceptionality

Example

$$\mathcal{T} = \{C \sqsubseteq \forall r.D\}$$

$$\mathcal{A} = \{C(a),\ r(a,b),\ \neg D(b)\}$$

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Typicality of relations not accounted for by existing approaches

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Focus restricted to (concept) subsumption and entailment

Beyond object typicality: semantics

Definition (r-ordered interpretation)

An r-ordered interpretation is a tuple $\mathcal{R} := \langle \Delta^{\mathcal{R}}, \cdot^{\mathcal{R}}, \prec^{\mathcal{R}} \rangle$ where

- $\langle \Delta^{\mathcal{R}}, \cdot^{\mathcal{R}} \rangle$ is a DL interpretation
- $\prec^{\mathcal{R}} := \langle \prec_1^{\mathcal{R}}, \ldots, \prec_n^{\mathcal{R}} \rangle$ where
  - $\prec_i^{\mathcal{R}} \subseteq r_i^{\mathcal{R}} \times r_i^{\mathcal{R}}$, for $1 \leq i \leq n$
  - Each $\prec_i^{\mathcal{R}}$ is a well-founded strict partial order on $r_i^{\mathcal{R}}$

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Intuition

- Pairs lower down in $\prec^{\mathcal{R}}$ are more normal than those higher up
- For each $R \subseteq r_i^{\mathcal{R}}$, $\min_{\prec_i^{\mathcal{R}}} R$: most normal r_i-pairs in R w.r.t. $\prec_i^{\mathcal{R}}$

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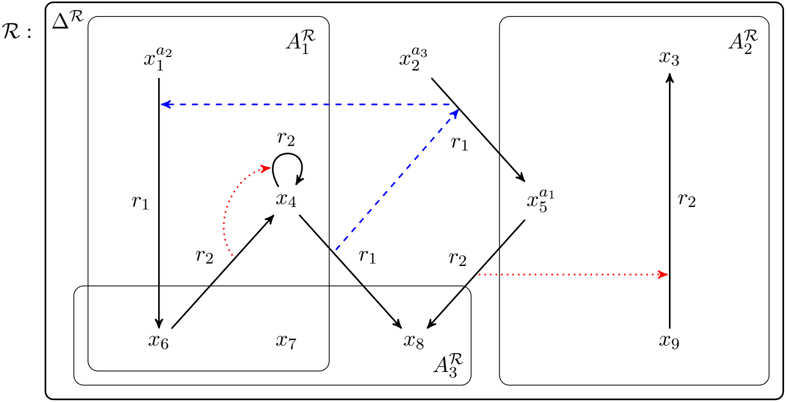
Notation

$$r_i^{\mathcal{R}|x} := r_i^{\mathcal{R}} \cap (\{x\} \times \Delta^{\mathcal{R}})$$

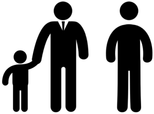
Beyond object typicality: semantics



- $\min_{\prec_1^{\mathcal{R}}} r_1^{\mathcal{R}} = \{x_4 x_8\}$
- $\min_{\prec_2^{\mathcal{R}}} r_2^{\mathcal{R}} = \{x_6 x_4, x_5 x_8\}$

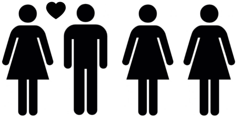
Beyond defeasible subsumption

Defeasible quantifiers



- $C ::= A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \forall r.C \mid \exists r.C \mid \widetilde{\forall} r.C \mid \widetilde{\exists} r.C$
- E.g.: $\widetilde{\forall}$guardianOf.Minor
- ‘Those individuals whose all normal guardianship relations are of minors’
- $(\widetilde{\forall} r_i.C)^{\mathcal{R}} := \{x \mid$ for all y, if $(x,y) \in \min_{\prec_i^{\mathcal{R}}}(r_i^{\mathcal{R}|x})$, then $y \in C^{\mathcal{R}}\}$

Defeasible number restrictions



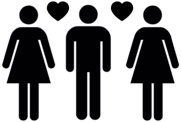
- $C ::= \cdots \mid \leq nr.C \mid \geq nr.C \mid \gtrsim nr.C \mid \lesssim nr.C$
- E.g.: $\lesssim 2$hasSibling.Female, ≈ 1marriedTo.$\top$
- In the latter, ‘those individuals in one normal marriage’
- $(\gtrsim nr_i.C)^{\mathcal{R}} := \{x \mid \#\{y \mid (x,y) \in \min_{\prec_i^{\mathcal{R}}}(r_i^{\mathcal{R}|x})$ and $y \in C^{\mathcal{R}}\} \geq n\}$

Beyond defeasible subsumption

Defeasible role inclusions

- $r_1 \mathrel{\underset{\sim}{\sqsubset}} r_2$
- E.g.: guardianOf $\mathrel{\underset{\sim}{\sqsubset}}$ parentOf
- ‘Usually, the role of guardianship is also that of being a parent’
- $\mathcal{R} \Vdash r_i \mathrel{\underset{\sim}{\sqsubset}} r_j$ iff $\min_{\prec_i^{\mathcal{R}}} r_i^{\mathcal{R}} \subseteq r_j^{\mathcal{R}}$

Defeasible role assertions



- Role properties that may fail
- E.g.: partOf is *usually transitive*, marriedTo is *normally functional*
- dFun(r_i), dTra(r_i), dDis(r_i, r_j), ...
- $\min_{\prec_i^{\mathcal{R}}} r_i^{\mathcal{R}}$ is functional, $\min_{\prec_i^{\mathcal{R}}} r_i^{\mathcal{R}}$ is transitive, $\min_{\prec_i^{\mathcal{R}}} r_i^{\mathcal{R}} \cap \min_{\prec_j^{\mathcal{R}}} r_j^{\mathcal{R}} = \emptyset$
- Alternative: $\top \sqsubseteq {\lesssim} 1r$ in the TBox or $r \circ r \mathrel{\underset{\sim}{\sqsubset}} r$ in the RBox

Beyond defeasible subsumption

Theorem

$\mathcal{ALC}$ extended with our new constructors is decidable

Theorem

Concept satisfiability and subsumption w.r.t. acyclic TBoxes: PSPACE-complete

Theorem

Concept satisfiability and subsumption w.r.t. general TBoxes: EXPTIME-complete

A glimpse of Defeasible $\mathcal{SROIQ}$

Concepts

- All $\mathcal{SROIQ}$ concepts
- $\forall\!\!\!\sim r.C$, $\exists\!\!\!\sim r.C$, $\exists\!\!\!\sim r.$Self
- $\gtrsim ns.C$, $\lesssim ns.C$ (s is a simple role)

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Statements and assertions

- All $\mathcal{SROIQ}$ statements and assertions
- $C \sqsubsetsim D$, $r \circ r \sqsubsetsim r$, $\mathsf{inv}(r) \sqsubsetsim r$
- $s_1 \circ \cdots \circ s_n \sqsubsetsim r$, $r \circ s_1 \circ \cdots \circ s_n \sqsubsetsim r$, $s_1 \circ \cdots \circ s_n \circ r \sqsubsetsim r$
- $\mathsf{dFun}(r)$, $\mathsf{dRef}(r)$, $\mathsf{dIrr}(r)$, $\mathsf{dSym}(r)$, $\mathsf{dAsy}(r)$, $\mathsf{dTra}(r)$, $\mathsf{dDis}(r,s)$

A glimpse of Defeasible $\mathcal{SROIQ}$

Reduction of $\langle \mathcal{T}, \mathcal{A}, \mathcal{R} \rangle$ to $\langle \emptyset, \emptyset, \mathcal{R} \rangle$

- Elimination of the ABox as for classical $\mathcal{SROIQ}$
- Rewriting of classical role assertions as for classical $\mathcal{SROIQ}$
- Rewriting of defeasible role assertions: Replace
  - dFun(r) with $\top \sqsubseteq \lesssim 1r.\top$
  - dRef(r) with $\top \mathrel{\sqsubset\!\!\!\sim} \exists r.$Self and dIrr$(r)$ with $\top \mathrel{\sqsubset\!\!\!\sim} \neg\exists r.$Self
  - dSym(r) with $r^{-} \mathrel{\sqsubset\!\!\!\sim} r$
  - dTra(r) with $r \circ r \mathrel{\sqsubset\!\!\!\sim} r$
  - dAsy(r) with dDis(r, r^{-})
- Rewrite $C \mathrel{\sqsubset\!\!\!\sim} D$ in $\mathcal{SROIQ}$
- Eliminate the TBox and the universal role as for classical $\mathcal{SROIQ}$

A glimpse of Defeasible $\mathcal{SROIQ}$

Tableau system

- Extension of the $\mathcal{SROIQ}$ tableau:
  - Rules $\sqcap$, $\sqcup$, $\forall$, ch, $\leq$ and o do not change (modulo new blocking and merging)
  - New versions of $\exists$-, Self-, $\geq$- and NN-rules
  - 8 new rules for defeasible constructs
- For now, we don't deal with role composition on the LHS of $\sqsubseteq\!\!\!\!\!\sim$

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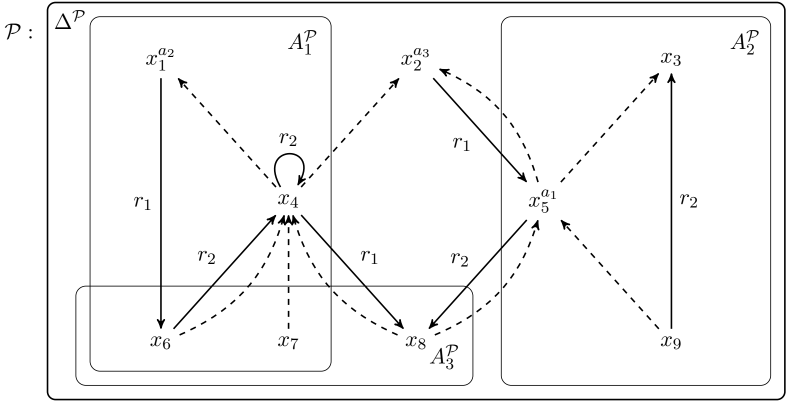
Theorem

Let C be a concept and $\mathcal{R}$ an RBox.

- The tableau algorithm terminates if started with nnf(C) and $\mathcal{R}$
- When exhaustively applied to nnf(C) and $\mathcal{R}$, the expansion rules yield a complete and clash-free completion graph iff C is satisfiable w.r.t. $\mathcal{R}$

Issues with a single ordering on objects



Assume

- Cheninblanc $\sqsubsetsim$ $\exists$hasAroma.Floral
- Cheninblanc $\sqsubsetsim$ $\forall$hasOrigin.Loire

Then

- No way to have $x \prec y$ and $y \prec x$
- No model of reality!

Contextual defeasible subsumption

$$C \precsim_r D$$

Definition

Let $\mathcal{R} = \langle \Delta^{\mathcal{R}}, \cdot^{\mathcal{R}}, \prec^{\mathcal{R}} \rangle$ be an r-ordered interpretation. For every role r, let

$$\begin{aligned} <_r^{\mathcal{R}} \ := \ \{(x,y) \mid \ & \text{for some } z,u \in \Delta^{\mathcal{R}}, [((x,z),(y,u)) \in \prec_r^{\mathcal{R}}] \text{ and} \\ & \text{for no } z,u \in \Delta^{\mathcal{R}}, [((y,u),(x,z)) \in \prec_r^{\mathcal{R}}]\} \end{aligned}$$

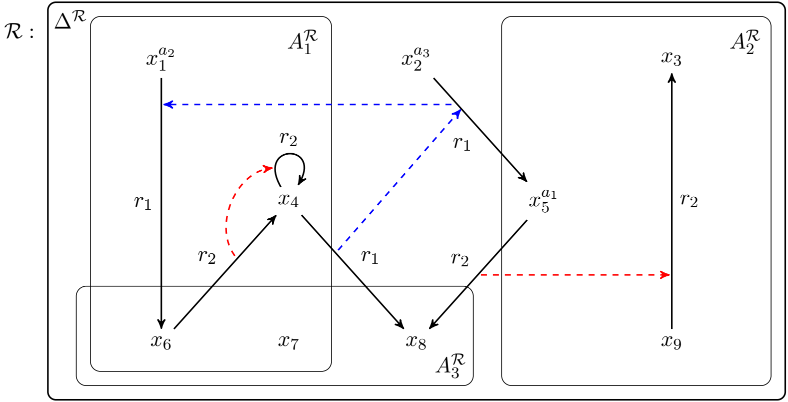
Then

- $\mathcal{R} \Vdash C \precsim_r D$ iff $\min_{<_r^{\mathcal{R}}} C^{\mathcal{R}} \subseteq D^{\mathcal{R}}$

Example

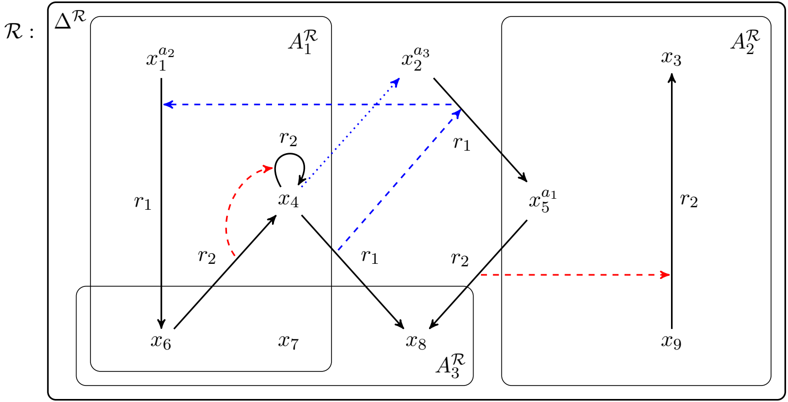
- Cheninblanc $\precsim_{\text{hasAroma}} \exists$hasAroma.Floral
- Cheninblanc $\precsim_{\text{hasOrigin}} \forall$hasOrigin.Loire

Contextual defeasible subsumption



- $\prec_1^{\mathcal{R}} = \{(x_4x_8, x_2x_5), (x_2x_5, x_1x_6), (x_4x_8, x_1x_6)\}$
- $\prec_2^{\mathcal{R}} = \{(x_5x_8, x_9x_3), (x_6x_4, x_4x_4)\}$
- $<_1^{\mathcal{R}} = \{(x_4, x_2), (x_2, x_1), (x_4, x_1)\}$
- $<_2^{\mathcal{R}} = \{(x_5, x_9), (x_6, x_4)\}$

Contextual defeasible subsumption



- $\prec_1^{\mathcal{R}} = \{(x_4x_8, x_2x_5), (x_2x_5, x_1x_6), (x_4x_8, x_1x_6)\}$
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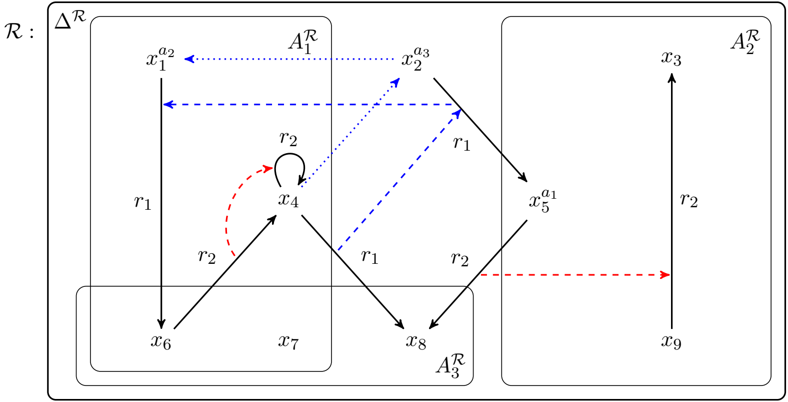

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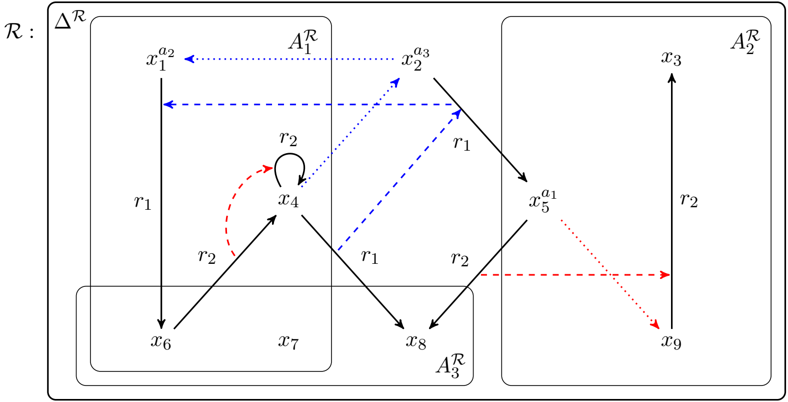

- $\prec_1^{\mathcal{R}} = \{(x_4x_8, x_2x_5), (x_2x_5, x_1x_6), (x_4x_8, x_1x_6)\}$
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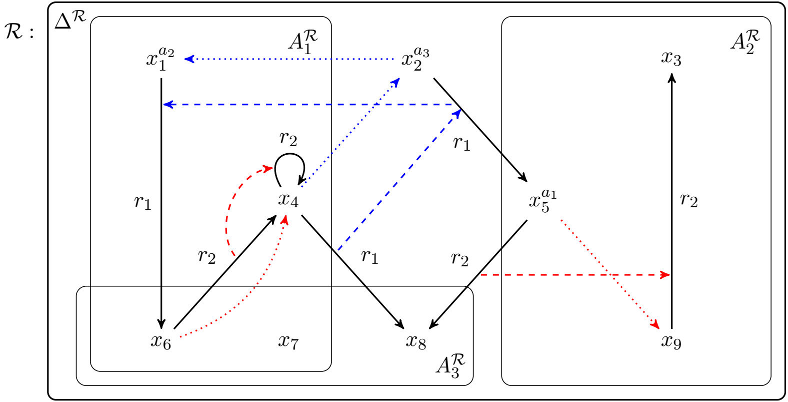

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Contextual defeasible subsumption

Lemma

For every r, $\sqsubset\!\!\!\!\sim_r$ is preferential (in the KLM sense). If $<_r^{\mathcal{R}}$ is a modular order, then $\sqsubset\!\!\!\!\sim_r$ is rational.

Contextual defeasible subsumption

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Moreover

- Contextual rational closure: generalisation to contexts (under review)
- But, of course, it still doesn't satisfy

$$(\exists\mathsf{M})\ \frac{C \sqsubseteq\!\!\!\!\sim_r D}{\exists r.C \sqsubseteq\!\!\!\!\sim_r \exists r.D}$$

$$(\forall\mathsf{M})\ \frac{C \sqsubseteq\!\!\!\!\sim_r D}{\forall r.C \sqsubseteq\!\!\!\!\sim_r \forall r.D}$$

Outline

Conclusion

Summary

- DLs are classical: no coping with exceptions and inconsistencies
- Defeasible subsumption, properties and entailment (and issues)
- A family of new defeasible constructors and their properties
- Theoretical feasibility of the proposed frameworks
- An approach to contextual defeasible subsumption

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Ongoing and future work

- Extensions of rational closure in the new framework
- Implementation of Protégé plugins followed by experimental tests
- Conjunctive-query answering in the richer languages

References

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Thank you!