Formal Foundations of Ontologies and Reasoning

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Outline of Part 2

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Making Statements

DL Knowledge Bases

Entailment in DLs

Motivation

Concept language of $\mathcal{ALC}$

$\top, \ \bot$	(constants)
A	(atomic concept)
$\neg C$	(complement of C)
$C \sqcap D$	(intersection of C and D)
$C \sqcup D$	(union of C and D)
$\exists r.C$	(existential restriction)
$\forall r.C$	(value restriction)

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Something is missing

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- What would C '$\rightarrow$' D mean here?

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Motivation

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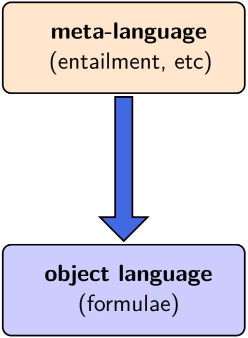
Something is missing

- The central notion in logic: C '$\rightarrow$' D
- What would C '$\rightarrow$' D mean here? (We already have $\neg C \sqcup D$)
- DLs have a version of '$\rightarrow$' that is very special

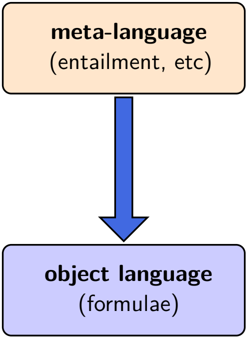
Statements

In many logics

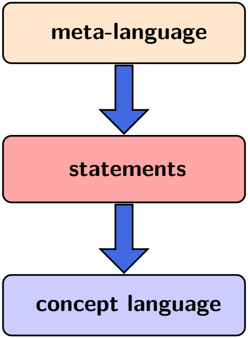


Statements

In many logics



In DLs



- Two levels of language
- Two notions of ‘entailment’
- Two notions of ‘satisfaction’

Making statements

Subsumption

- Concept inclusion
- Employed students are students
- Employed students are employees

Making statements

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Instantiation or assertions

- Concept and role membership
- John is an employed student (John instantiates employed student)
- John works for IBM (John and IBM instantiate works for)

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Statements talk about concepts, roles and individuals

Making statements

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Instantiation or assertions

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Statements talk about concepts, roles and individuals

They are not concepts! They are in the 'in-between' language

Subsumption statements

$$C \sqsubseteq D$$

Intuition

- D subsumes C (or C is subsumed by D)
- C is more specific than D (or D is more general than C)
- Formalise one aspect of is-a relations

Subsumption statements

$$C \sqsubseteq D$$

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- Formalise one aspect of is-a relations

Example

- $\mathsf{EmpStud} \sqsubseteq \mathsf{Student} \sqcap \mathsf{Employee}$, $\mathsf{Employee} \sqsubseteq \exists \mathsf{worksFor}.\top$
- $\mathsf{EmpStud} \sqsubseteq \exists \mathsf{pays}.\mathsf{Tax}$, $\mathsf{Student} \sqcap \neg \mathsf{Employee} \sqsubseteq \neg \exists \mathsf{pays}.\mathsf{Tax}$

Subsumption statements

$$C \sqsubseteq D$$

Intuition

- D subsumes C (or C is subsumed by D)
- C is more specific than D (or D is more general than C)
- Formalise one aspect of is-a relations

Example

- $\text{EmpStud} \sqsubseteq \text{Student} \sqcap \text{Employee}$, $\text{Employee} \sqsubseteq \exists \text{worksFor}.\top$
- $\text{EmpStud} \sqsubseteq \exists \text{pays}.\text{Tax}$, $\text{Student} \sqcap \neg\text{Employee} \sqsubseteq \neg\exists \text{pays}.\text{Tax}$

Central notion in DL terminologies (taxonomies)

Subsumption statements

$$C \sqsubseteq D$$

Semantics

- $\mathcal{I} \Vdash C \sqsubseteq D$ ($\mathcal{I}$ satisfies $C \sqsubseteq D$) if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
- First level of 'entailment': all C-objects are D-objects

Subsumption statements

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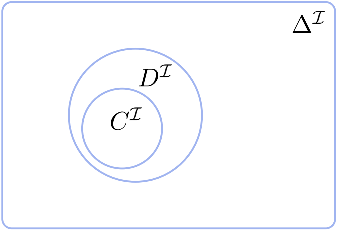

Subsumption statements

$$C \equiv D$$

Concept equivalence

- Just an abbreviation for $C \sqsubseteq D$ and $D \sqsubseteq C$
- $\mathcal{I} \Vdash C \equiv D$ if $\mathcal{I} \Vdash C \sqsubseteq D$ and $\mathcal{I} \Vdash D \sqsubseteq C$
- $\mathcal{I} \Vdash C \equiv D$ if $C^{\mathcal{I}} = D^{\mathcal{I}}$

Subsumption statements

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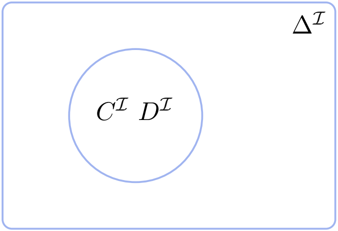
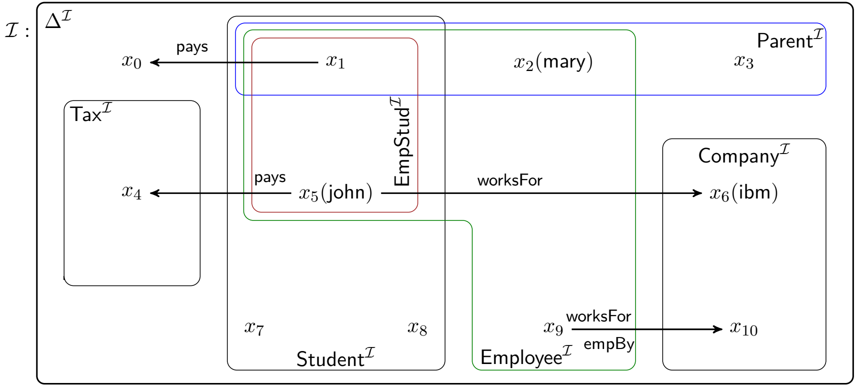
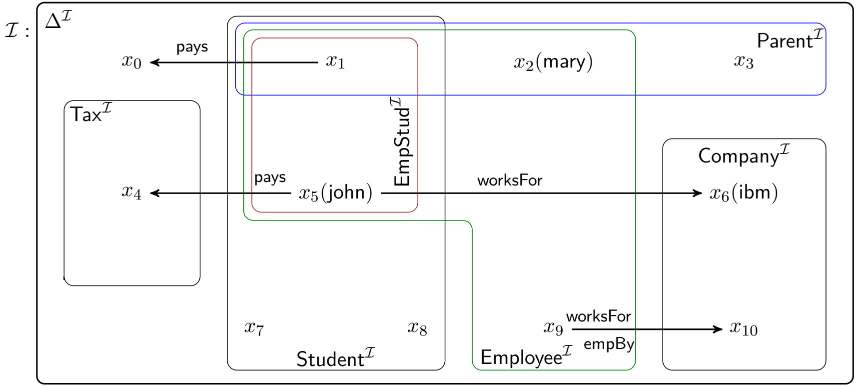


Exercise

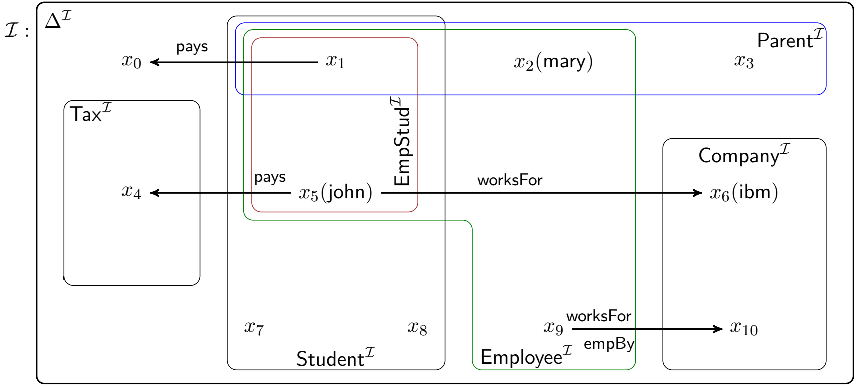

$\mathcal{I}$:
$\Delta^{\mathcal{I}}$
pays
x_0
x_1
x_2(mary)
x_3
Parent$^{\mathcal{I}}$
Tax$^{\mathcal{I}}$
EmpStud$^{\mathcal{I}}$
Company$^{\mathcal{I}}$
pays
worksFor
x_4
x_5(john)
x_6(ibm)
worksFor
x_7
x_8
x_9
x_{10}
empBy
Student$^{\mathcal{I}}$
Employee$^{\mathcal{I}}$

Exercise



- $\mathcal{I} \Vdash \text{EmpStud} \sqsubseteq \text{Student} \sqcap \text{Employee}$?
- $\mathcal{I} \Vdash \exists\text{worksFor}.\top \sqsubseteq \text{Employee}$?
- $\mathcal{I} \Vdash \text{Employee} \sqsubseteq \exists\text{worksFor}.\top$?
- $\mathcal{I} \Vdash \text{EmpStud} \sqsubseteq \forall\text{pays}.\text{Tax}$?
- $\mathcal{I} \Vdash \text{Parent} \sqcap \neg\text{Employee} \sqsubseteq \text{Tax} \sqcup \neg\text{Student}$?
- $\mathcal{I} \Vdash \text{EmpStud} \sqcap \text{Parent} \sqsubseteq \exists\text{pays}.\text{Tax}$?
- $\mathcal{I} \Vdash \exists\text{empBy}.\top \sqsubseteq \exists\text{worksFor}.\text{Company}$?
- $\mathcal{I} \Vdash \forall\text{pays}.\text{Tax} \sqsubseteq \exists\text{empBy}.\text{Company}$?

Exercise



- $\mathcal{I} \Vdash \mathsf{EmpStud} \sqsubseteq \mathsf{Student} \sqcap \mathsf{Employee}$ Yep!
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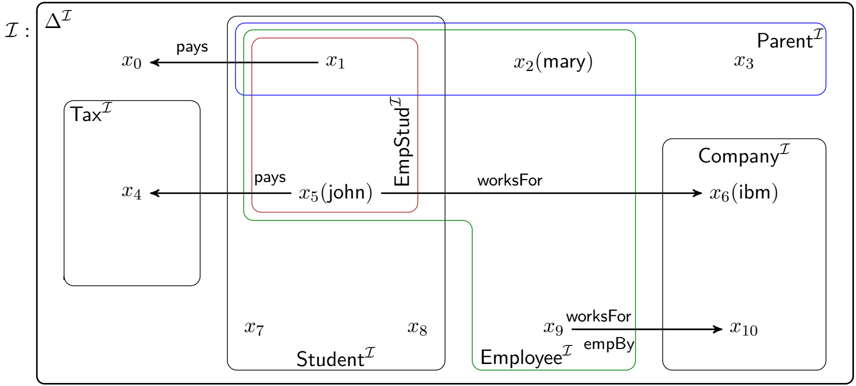
Exercise



- $\mathcal{I} \Vdash$ EmpStud $\sqsubseteq$ Student $\sqcap$ Employee Yep!
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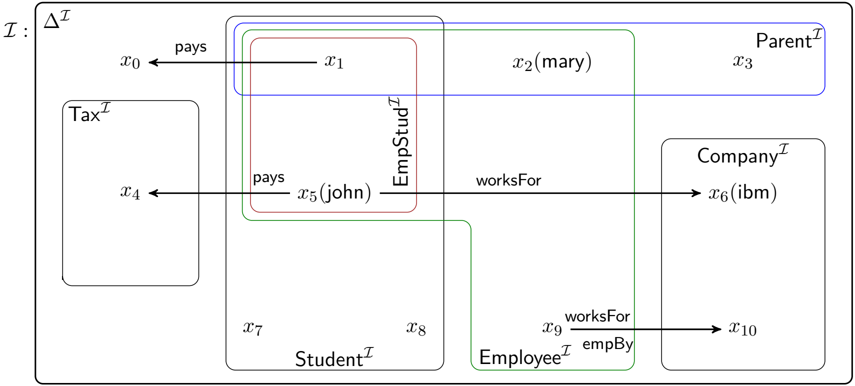

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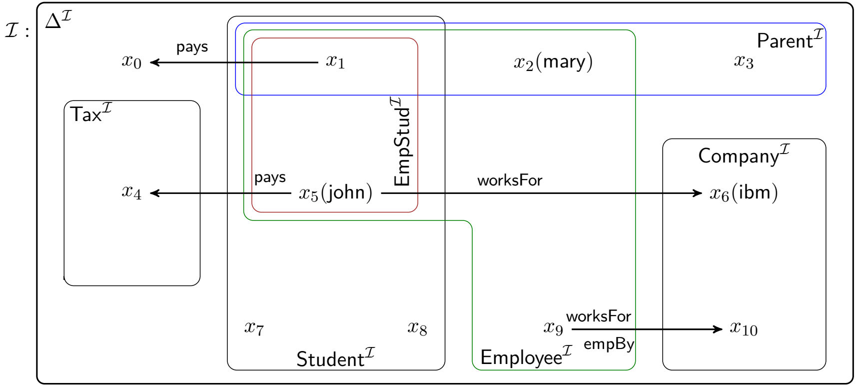

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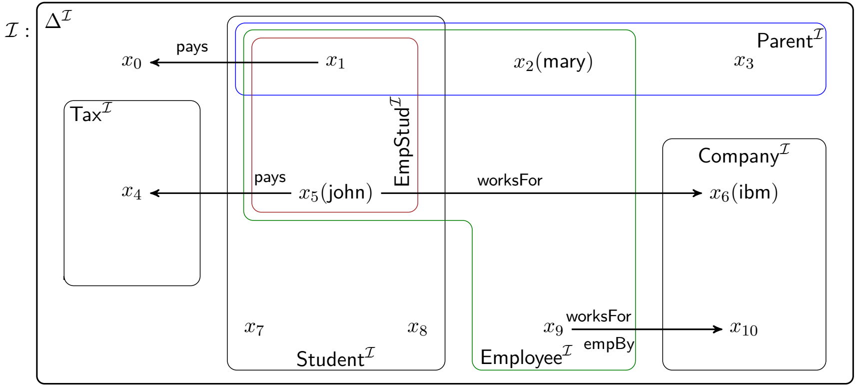

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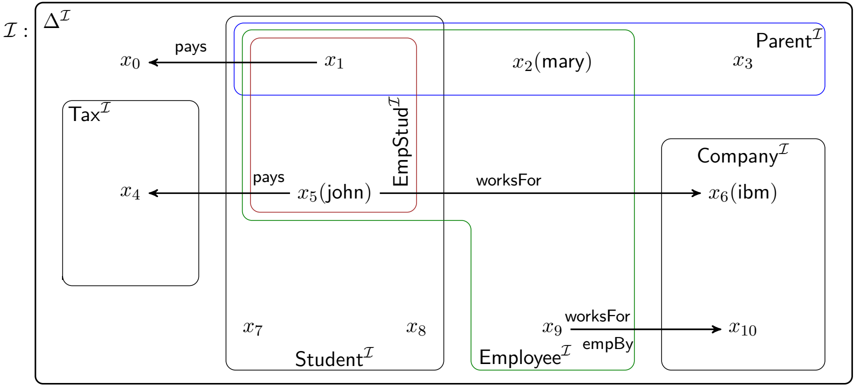

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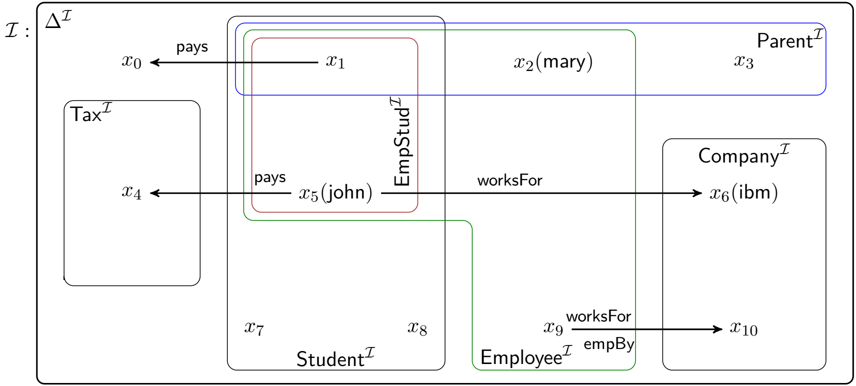

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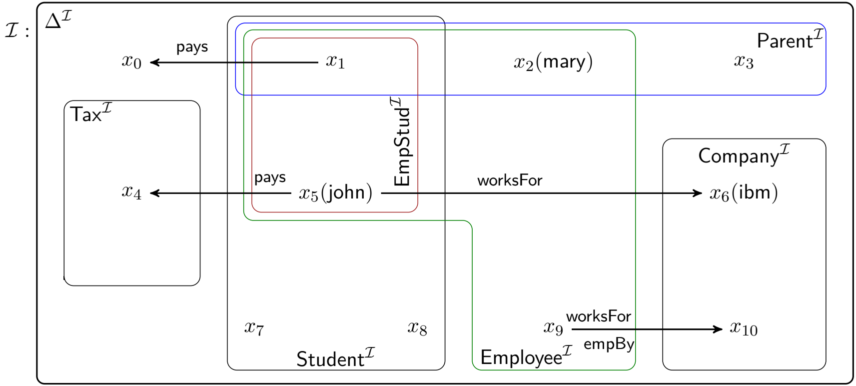

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Assertions

$$a : C \qquad (a, b) : r$$

Intuition

- a is an instance of C
- a and b are related via r (or (a, b) is an instance of r)
- Formalise another aspect of is-a relations

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Example

- $\mathsf{john} : \mathsf{EmpStud}$, $\mathsf{mary} : \mathsf{Parent} \sqcap \neg\exists \mathsf{worksFor}.\top \sqcap \neg\exists \mathsf{pays}.\mathsf{Tax}$
- $(\mathsf{john}, \mathsf{ibm}) : \mathsf{worksFor}$

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- $(\mathsf{john}, \mathsf{ibm}) : \mathsf{worksFor}$ (in some DLs: $(\mathsf{john}, \mathsf{ibm}) : \neg\mathsf{empBy}$)

Assertions

$$a : C \qquad (a, b) : r$$

Intuition

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- a and b are related via r (or (a, b) is an instance of r)
- Formalise another aspect of is-a relations

Example

- john : EmpStud, mary : Parent $\sqcap \neg\exists$worksFor.$\top \sqcap \neg\exists$pays.Tax
- (john, ibm) : worksFor (in some DLs: (john, ibm) : $\neg$empBy)

Central notion in DL 'databases'

Assertions

$$a : C \qquad\qquad (a, b) : r$$

Semantics

- $\mathcal{I} \Vdash a : C$ ($\mathcal{I}$ satisfies $a : C$) if $a^{\mathcal{I}} \in C^{\mathcal{I}}$
- $\mathcal{I} \Vdash (a, b) : r$ ($\mathcal{I}$ satisfies $(a, b) : r$) if $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$
- First level of 'satisfaction': a is a 'model' of C, (a, b) is a 'model' of r

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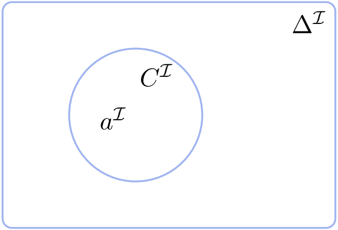
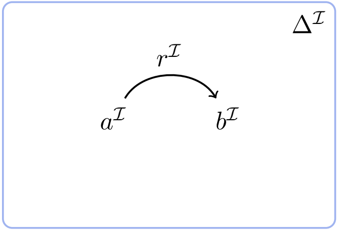
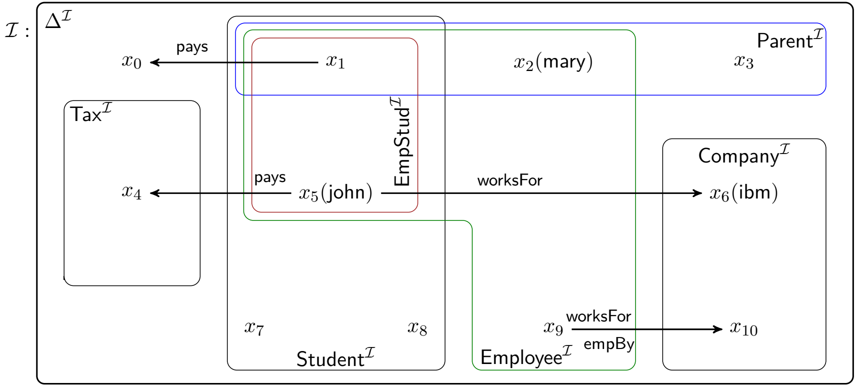
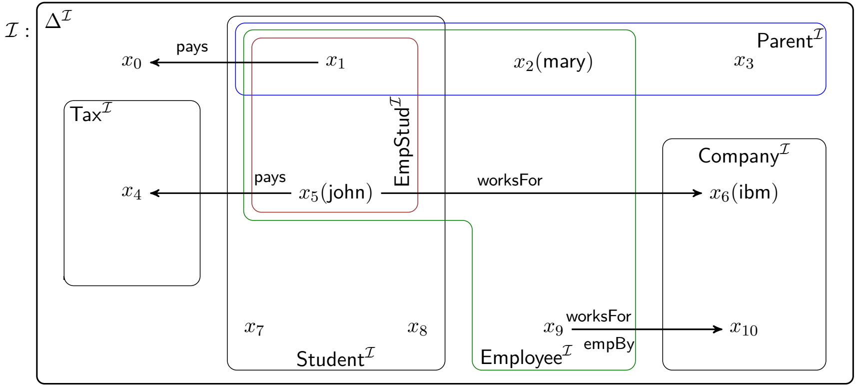




Exercise

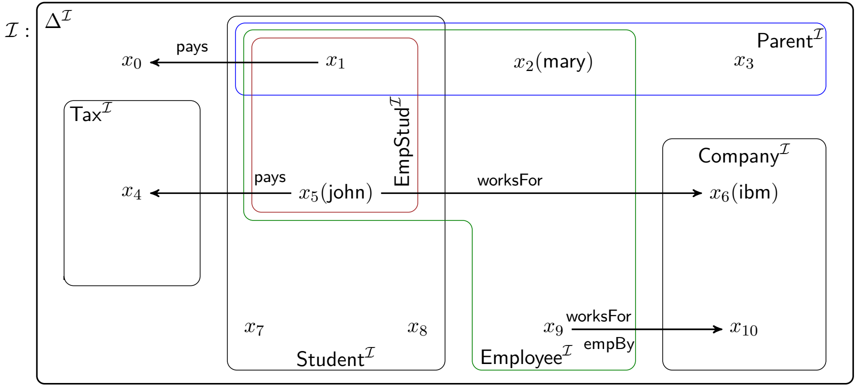

$\mathcal{I}$:
$\Delta^{\mathcal{I}}$
pays
x_0
x_1
x_2(mary)
x_3
Parent$^{\mathcal{I}}$
Tax$^{\mathcal{I}}$
EmpStud$^{\mathcal{I}}$
Company$^{\mathcal{I}}$
pays
x_4
x_5(john)
worksFor
x_6(ibm)
x_7
x_8
x_9
worksFor
empBy
x_{10}
Student$^{\mathcal{I}}$
Employee$^{\mathcal{I}}$

Exercise



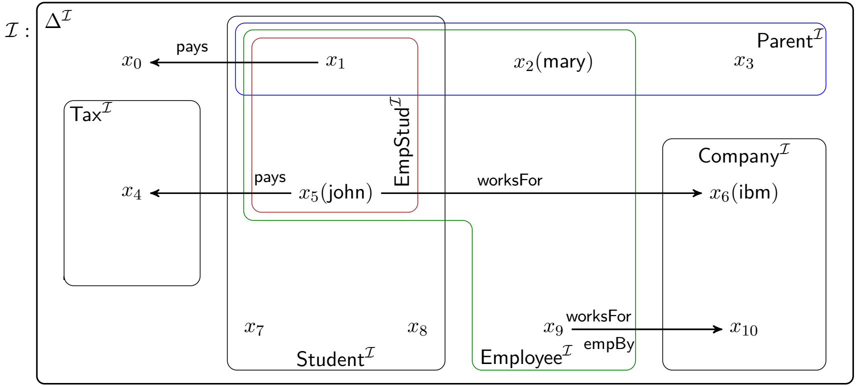
- $\mathcal{I} \Vdash$ john : Employee $\sqcap$ $\exists$pays.Tax ?
- $\mathcal{I} \Vdash$ mary : $\forall$pays.Tax ?
- $\mathcal{I} \Vdash$ (mary, ibm) : empBy ?
- $\mathcal{I} \Vdash$ (ibm, john) : worksFor ?
- $\mathcal{I} \Vdash$ mary : Parent $\sqcap$ $\neg\exists$worksFor.$\top$ $\sqcap$ $\neg\exists$pays.Tax ?
- $\mathcal{I} \Vdash$ mary : Employee $\sqcap$ $\exists$empBy.$\top$?
- $\mathcal{I} \Vdash$ john : $\forall$empBy.Company ?
- $\mathcal{I} \Vdash$ john : $\exists$worksFor.$\forall$pays.$\bot$?

Exercise



- $\mathcal{I} \Vdash \text{john} : \text{Employee} \sqcap \exists \text{pays}.\text{Tax}$ Yep!
- $\mathcal{I} \Vdash \text{mary} : \forall \text{pays}.\text{Tax}$?
- $\mathcal{I} \Vdash (\text{mary}, \text{ibm}) : \text{empBy}$?
- $\mathcal{I} \Vdash (\text{ibm}, \text{john}) : \text{worksFor}$?
- $\mathcal{I} \Vdash \text{mary} : \text{Parent} \sqcap \neg\exists \text{worksFor}.\top \sqcap \neg\exists \text{pays}.\text{Tax}$?
- $\mathcal{I} \Vdash \text{mary} : \text{Employee} \sqcap \exists \text{empBy}.\top$?
- $\mathcal{I} \Vdash \text{john} : \forall \text{empBy}.\text{Company}$?
- $\mathcal{I} \Vdash \text{john} : \exists \text{worksFor}.\forall \text{pays}.\bot$?

Exercise



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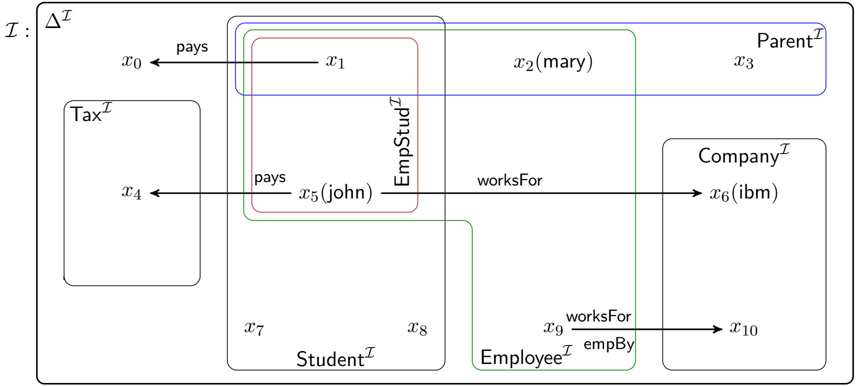

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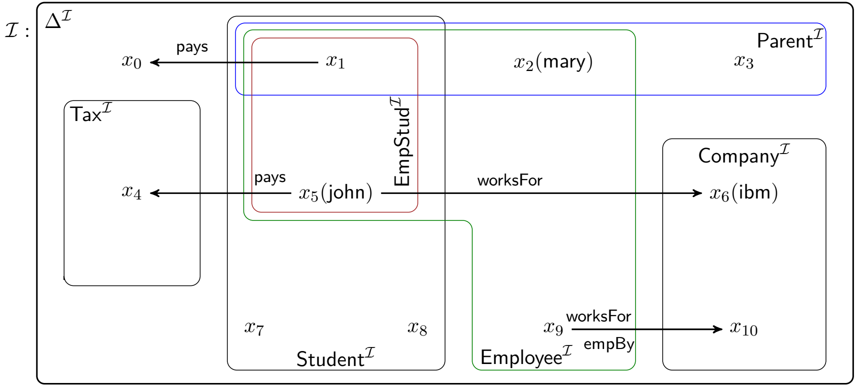

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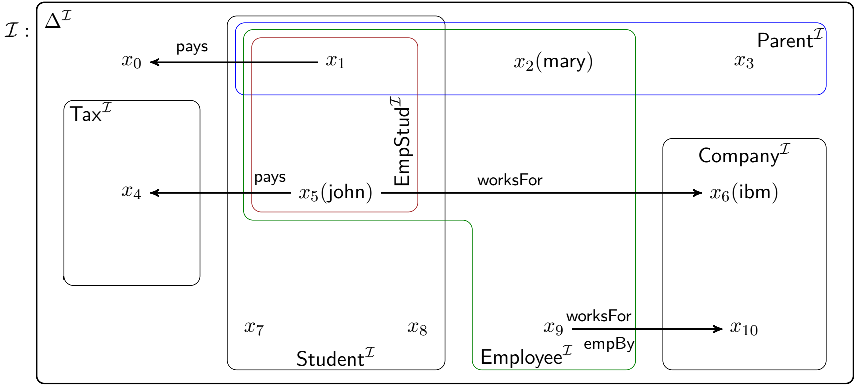

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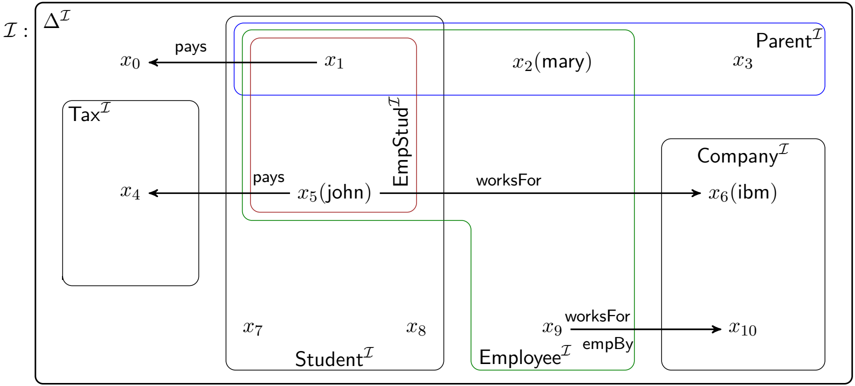

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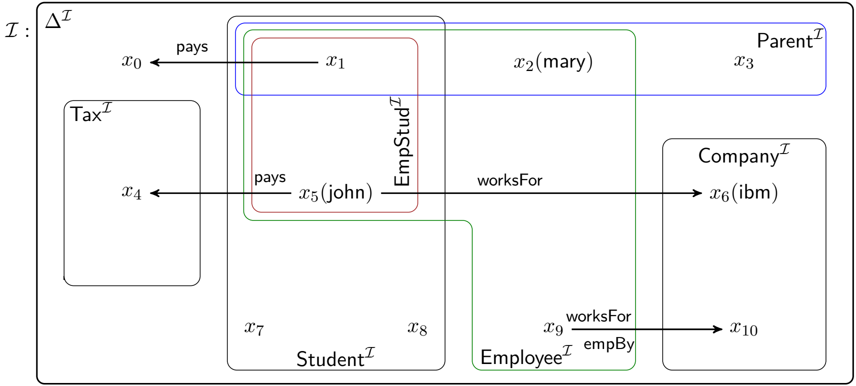

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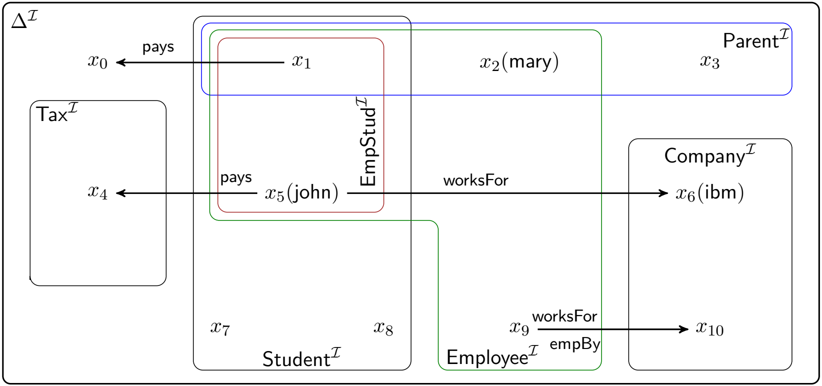

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Validity

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- $\models \neg(C \sqcup D) \equiv (\neg C \sqcap \neg D)$
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Watch out: Statements can be valid; concepts cannot!

Outline of Part 2

Making Statements

DL Knowledge Bases

Entailment in DLs

TBoxes and ABoxes

Intensional knowledge

- Set of subsumption statements
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Definition (Knowledge base)

A DL knowledge base (a.k.a. ontology) is a tuple $\mathcal{KB} =_{\text{def}} \langle \mathcal{T}, \mathcal{A} \rangle$

Knowledge bases

Example (The student ontology in DL)

$$\mathcal{T} = \left\{ \begin{array}{c} \mathsf{EmpStud} \equiv \mathsf{Student} \sqcap \mathsf{Employee}, \\ \mathsf{Student} \sqcap \neg \mathsf{Employee} \sqsubseteq \neg \exists \mathsf{pays}.\mathsf{Tax}, \\ \mathsf{EmpStud} \sqcap \neg \mathsf{Parent} \sqsubseteq \exists \mathsf{pays}.\mathsf{Tax}, \\ \mathsf{EmpStud} \sqcap \mathsf{Parent} \sqsubseteq \neg \exists \mathsf{pays}.\mathsf{Tax}, \\ \exists \mathsf{worksFor}.\mathsf{Company} \sqsubseteq \mathsf{Employee} \end{array} \right\}$$

$$\mathcal{A} = \left\{ \begin{array}{c} \mathsf{ibm} : \mathsf{Company}, \\ \mathsf{mary} : \mathsf{Parent}, \\ \mathsf{john} : \mathsf{EmpStud}, \\ (\mathsf{john}, \mathsf{ibm}) : \mathsf{worksFor} \end{array} \right\}$$

classes
relations
individuals

Knowledge bases

Semantics

- $\mathcal{I} \Vdash \mathcal{T}$ if $\mathcal{I} \Vdash C \sqsubseteq D$ for every $C \sqsubseteq D \in \mathcal{T}$
- $\mathcal{I} \Vdash \mathcal{A}$ if:
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Moreover

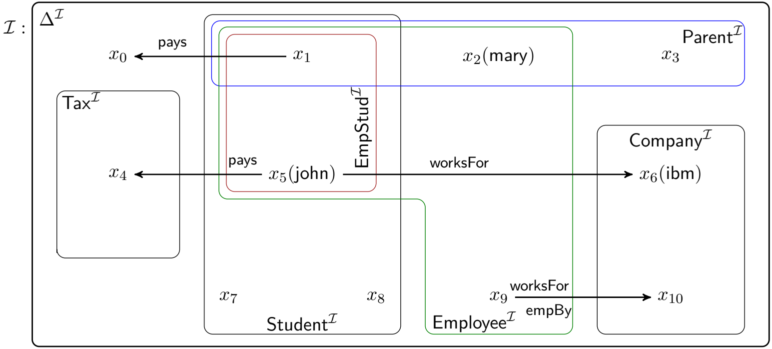
- If $\mathcal{I} \Vdash \mathcal{T}$, we say $\mathcal{I}$ is a model of $\mathcal{T}$
- If $\mathcal{I} \Vdash \mathcal{A}$, we say $\mathcal{I}$ is a model of $\mathcal{A}$
- If $\mathcal{I} \Vdash \mathcal{T} \cup \mathcal{A}$, then $\mathcal{I}$ is a model of $\mathcal{KB} = \langle \mathcal{T}, \mathcal{A} \rangle$
- $\mathcal{KB}$ is satisfiable if it has a model

Exercise

- Let $\mathcal{KB} = \langle \mathcal{T}, \mathcal{A} \rangle$, where:

$$\mathcal{T} = \left\{ \begin{array}{c} \mathsf{EmpStud} \equiv \mathsf{Student} \sqcap \mathsf{Employee}, \\ \mathsf{Student} \sqcap \neg\mathsf{Employee} \sqsubseteq \neg\exists\mathsf{pays}.\mathsf{Tax}, \\ \mathsf{EmpStud} \sqcap \neg\mathsf{Parent} \sqsubseteq \exists\mathsf{pays}.\mathsf{Tax}, \\ \mathsf{EmpStud} \sqcap \mathsf{Parent} \sqsubseteq \neg\exists\mathsf{pays}.\mathsf{Tax}, \\ \exists\mathsf{worksFor}.\mathsf{Company} \sqsubseteq \mathsf{Employee} \end{array} \right\} \qquad \mathcal{A} = \left\{ \begin{array}{c} \mathsf{ibm} : \mathsf{Company}, \\ \mathsf{mary} : \mathsf{Parent}, \\ \mathsf{john} : \mathsf{EmpStud}, \\ (\mathsf{john}, \mathsf{ibm}) : \mathsf{worksFor} \end{array} \right\}$$

- Is the interpretation $\mathcal{I}$ below a model of $\mathcal{KB}$?



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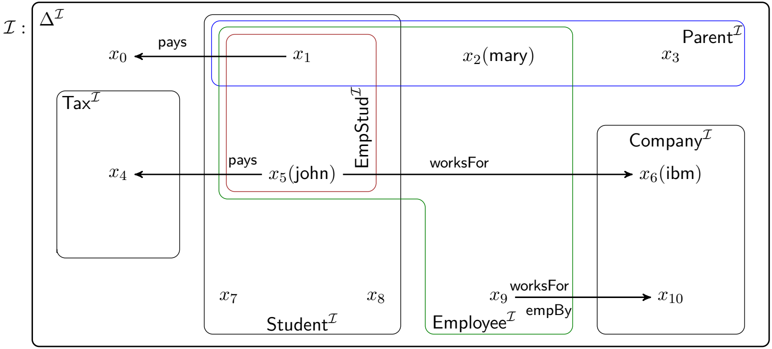

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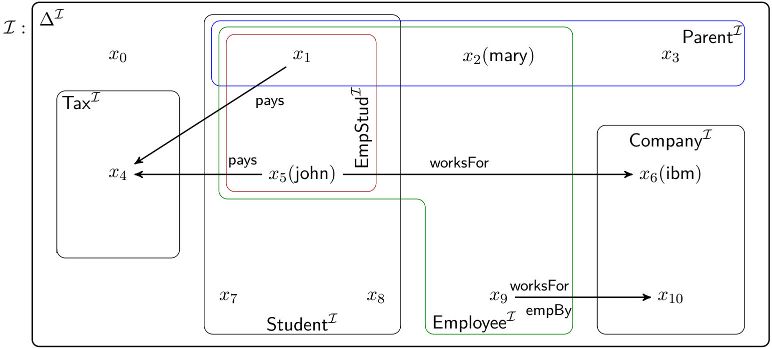

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What does follow from a KB?

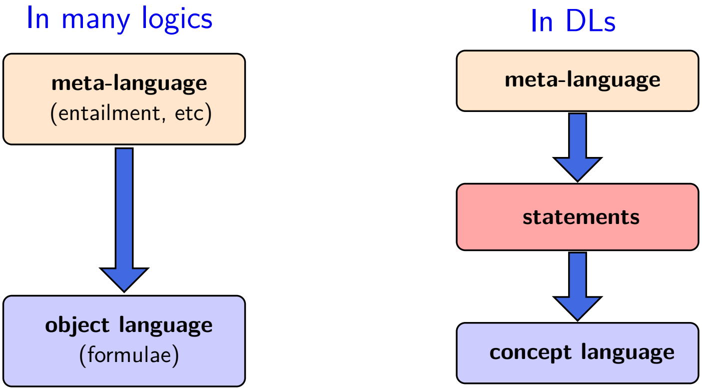
Example

- Is an employed parent an employee who is also a student?
- Is being an employee the same as being employed by someone?
- Is Mary an employed parent?
- Is being an employee more general than being a employed student?
- Is there anybody who is a student and a parent at the same time?
- Is my knowledge base consistent?
- Tell me, briefly, what John is.
- Who are the employed students who work for companies?

What does follow from a KB?

Entailment from KBs

- Defined on the level of statements (not concepts)
- Remember:



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Obvious definition of entailment

- $\mathcal{T} \models \alpha$ if $\mathcal{I} \Vdash \alpha$ for every $\mathcal{I}$ s.t. $\mathcal{I} \Vdash \mathcal{T}$
- $\mathcal{A} \models \alpha$ if $\mathcal{I} \Vdash \alpha$ for every $\mathcal{I}$ s.t. $\mathcal{I} \Vdash \mathcal{A}$
- $\mathcal{KB} \models \alpha$ if $\mathcal{I} \Vdash \alpha$ for every $\mathcal{I}$ s.t. $\mathcal{I} \Vdash \mathcal{KB}$

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- $\mathcal{KB} \models \mathsf{Student} \sqcap \exists\mathsf{worksFor}.\mathsf{Company} \sqcap \neg\mathsf{Parent} \sqsubseteq \mathsf{EmpStud} \sqcap \exists\mathsf{pays}.\mathsf{Tax}$
- $\mathcal{KB} \models \mathsf{john} : \mathsf{Student} \sqcap \exists\mathsf{worksFor}.\mathsf{Company}$
- $\mathcal{KB} \not\models \mathsf{mary} : \neg\exists\mathsf{pays}.\mathsf{Tax}$
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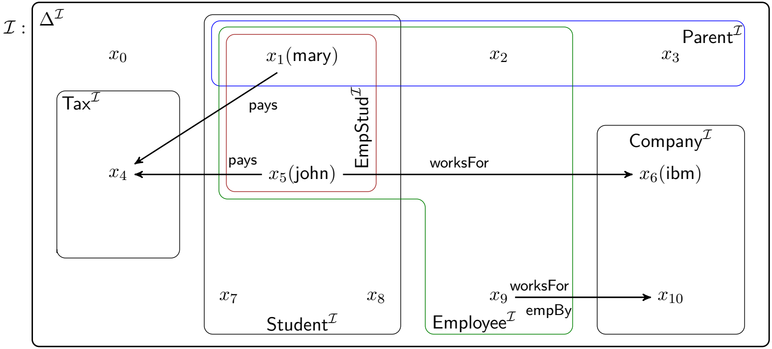

Open- v. closed-world assumption

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Example

$$\{(\mathsf{john}, \mathsf{ibm}) : \mathsf{worksFor},\ \mathsf{ibm} : \mathsf{Company})\} \models \mathsf{john} : \forall \mathsf{worksFor}.\mathsf{Company} \quad ?$$

- In Prolog: "Yep!"
- In DL-based systems: "Uh, I don't know . . . "

Epilogue

Summary

- Intensional and extensional knowledge
- Specifying DL knowledge bases
  - TBox: categories
  - ABox: partial view of the world
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What next?

- Reasoning with DL ontologies