A Note on a Defeasible Andi-Style Multi-Modal Logic of Actions

Ivan Varzinczak
Université Sorbonne Paris Nord
Inserm, Sorbonne Université, Limics, 93017 Bobigny, France
CAIR, University of Cape Town, Cape Town, South Africa
CNR–ISTI, Pisa, Italy
ivan.varzinczak@sorbonne-paris-nord.fr

Abstract

In this little homage to Andreas Herzig (Andi), I revisit a research topic I had the privilege of working on with him, namely modal-based approaches to *reasoning about actions*. Taking Andi's modal logics of action as a point of departure, I show how action domain descriptions can benefit from a deal of work done in the defeasible-reasoning literature to account for both exception tolerance and a commonly accepted notion of rationality in reasoning. The resulting logical framework is a more robust and resilient action formalism for reasoning about dynamic domains.

1 Introduction

Andreas Herzig (Andi) has been one of the pioneers endorsing modal logic [8] in general and dynamic logic [11] in particular as viable alternatives to first-order based formalisms, such as the situation calculus [18], for reasoning about actions, planning, and beyond. Modal logic has a syntax and a semantics that are both simpler and neater compared to those of first-order languages, and it lends itself naturally to the formalisation of many aspects of human knowledge and reasoning without excessive clumsiness. Additionally, modal logic is generally a decidable formalism, with many off-the-shelf algorithms and tools made available by the community over the past decades. These are features that have

always been of paramount importance to Andi for the practical use of logic and that have guided most of his work.

An Andi-style logic for reasoning about actions is a logical language with the following main features: (*i*) its syntax is a useful and elegant fragment of some modal system; (*ii*) it is expressive enough to allow for the specification of the different types of laws or rules associated with dynamic scenarios, including effect laws, executability and inexecutability laws, besides integrity constraints; (*iii*) it can be endowed with an intuitive and effective solution to the frame and ramification problems, and (*iv*) it can be equipped with a decision procedure for performing the various reasoning services associated with action domains.

A somewhat tacit tradition in the reasoning about actions literature has often been that the above-mentioned laws in general, but integrity constraints in particular, are hard constraints and, as a result, do not admit exceptions. Such is the case for Andi-style action domain descriptions, as modal sentences with which the various laws are formalised behave classically. Nevertheless, as widely investigated by the non-monotonic reasoning community, rules are prone to have exceptions, and systems capable of handling them are more robust and resilient.

The goal of this paper is to show how Andi-style action domain descriptions can be made more refined, tolerant to exceptions, and also more venturesome when it comes to reasoning. Building on recent work on defeasible reasoning for logics that are more expressive than propositional logic, in particular modal logic, we revisit Andi-style multi-modal logics of action by enriching them with defeasibility features, in particular with what is commonly called rationality at the entailment level. The resulting framework is a more robust and resilient action formalism.

The plan of the paper is as follows: Section 2 recalls the terminology and notation we use in the upcoming sections. In Section 3, we show how Andi-style action descriptions can be endowed with defeasible laws, of which a rational semantics borrowed from the defeasible description logic case [2] is given in Section 4. In Section 5, we equip our framework with a notion of entailment which has been acknowledged as suitable in other logics, namely the *rational closure* of a defeasible domain description. Section 6 concludes the paper with a discussion on further features of the framework here proposed and possible extensions thereof.

2 Preliminaries and notation

We assume a multi-modal language generated from a non-empty and finite set of propositional atoms $\mathcal{P}$, with the special constants $\top$ and $\bot$, and a finite set of (atomic) action names $\mathcal{A}$. We use $p, q, \ldots$ as meta-variables for atoms, and $a, b, \ldots$ to denote actions. Complex sentences are denoted by $\alpha, \beta, \ldots$, and are recursively defined by the grammar: $\alpha ::= \top \mid \bot \mid p \mid \neg\alpha \mid (\alpha \wedge \alpha) \mid (\alpha \vee \alpha) \mid (\alpha \rightarrow \alpha) \mid \Diamond_a\alpha \mid \Box_a\alpha$. With $\mathcal{L}$ we denote the set of all sentences of the underlying modal language. When writing down sentences of $\mathcal{L}$, we follow the usual convention and omit parentheses whenever they are not essential for disambiguation.

The semantics of $\mathcal{L}$ is the standard Kripkean one. A Kripke model is a structure $\mathscr{M} = \langle W, R, V\rangle$, where $W \neq \emptyset$ is a (possibly infinitely) countable set of *worlds*, $R \stackrel{\text{def}}{=} \langle R_a \mid a \in \mathcal{A}\rangle$, where each $R_a \subseteq W \times W$, $a \in \mathcal{A}$, is an *accessibility relation*, and $V\colon W \longrightarrow \{0,1\}^{\mathcal{P}}$ is a function mapping worlds into propositional valuations. Whenever it eases presentation, we shall represent valuations as sequences of 0s and 1s.

Sentences of $\mathcal{L}$ are true or false relative to a world in a Kripke model. For every w in $\mathscr{M}$: $\mathscr{M}, w \Vdash \top$; $\mathscr{M}, w \nVdash \bot$; $\mathscr{M}, w \Vdash p$ if $V(w)(p) = 1$; $\mathscr{M}, w \Vdash \neg\alpha$ if $\mathscr{M}, w \nVdash \alpha$; $\mathscr{M}, w \Vdash \alpha \wedge \beta$ if $\mathscr{M}, w \Vdash \alpha$ and $\mathscr{M}, w \Vdash \beta$; $\mathscr{M}, w \Vdash \Diamond_a\alpha$ if $\mathscr{M}, w' \Vdash \alpha$ for some w' s.t. $(w, w') \in R_a$, and $\mathscr{M}, w \Vdash \Box_a\alpha$ if $\mathscr{M}, w' \Vdash \alpha$ for all w' s.t. $(w, w') \in R_a$. Truth conditions for the other connectives are as usual. Given $\mathscr{M} = \langle W, R, V\rangle$ and $\alpha \in \mathcal{L}$, with $[\![\alpha]\!]^{\mathscr{M}} \stackrel{\text{def}}{=} \{w \in W \mid \mathscr{M}, w \Vdash \alpha\}$ we denote the *α-models* in $\mathscr{M}$.

3 Defeasible action domain descriptions

When specifying an action domain description, one usually writes down a set of 'rule'-like statements in the underlying logical language. These are commonly called *laws* and their purpose is to capture the behaviour of the actions as well as the structure of the domain under consideration.

Integrity constraints (ICs), also called *static laws*, are meant to ensure the structure of the world remains coherent as actions are executed. In a propositional modal setting as the one we assume here, they amount to propositional sentences, often in the form of a material implication, understood as *global axioms*. An example of IC is $\mathsf{walking} \rightarrow \mathsf{alive}$.

As it turns out, just like rules may fail or have exceptions, so do ICs, in particular if they do not encode some (rigid) laws of physics: a turkey whose head has just been chopped off but is still moving around, even if for a short while, ought not to be seen as alive anymore.

A *defeasible integrity constraint* (DIC) is a statement of the form $\alpha \mathrel{|\!\sim} \beta$, where α and β are *propositional* sentences, and is read as "usually, if α, then β." An example of a DIC is $\mathsf{walking} \mathrel{|\!\sim} \mathsf{alive}$, stating that usually, a walking turkey is alive. With a DIC $\alpha \mathrel{|\!\sim} \beta$, the intention is to capture the fact that a constraint expressed as a material implication of the form $\alpha \rightarrow \beta$ usually holds, but may still fail in exceptional circumstances. In our example, $\mathsf{walking} \mathrel{|\!\sim} \mathsf{alive}$ can accommodate the above exception.

Effect laws are statements capturing the most relevant aspects of an action's behaviour. In our setting, they are specified as a ('rule'-like) sentence of the form $\alpha \rightarrow \Box_a \beta$, with α, β propositional. For example, $\mathsf{loaded} \rightarrow \Box_{\mathsf{shoot}} \neg \mathsf{alive}$ links the precondition (the gun is loaded) to the effect (the turkey is dead) of the action in question (to shoot).

Actions may fail to produce their expected outcome: situations in which, e.g., the gun is presumably loaded but the bullet is stuck in its barrel and, as a result, the turkey keeps on being alive after shooting, violate the corresponding effect law. A *defeasible effect law* (DEL) is a rule-like statement of the form $\alpha \mathrel{|\!\sim} \Box_a \beta$, with α and β *propositional* sentences and $a \in \mathcal{A}$, and is read as "usually, if α, then after every execution of action a, β holds." As an example, we have $\mathsf{loaded} \mathrel{|\!\sim} \Box_{\mathsf{shoot}} \neg \mathsf{alive}$. Intuitively, such a statement captures the expected effect of shooting in normal situations (the turkey's death) while allowing for (less normal) outcomes as the one we referred to above.

A special type of effect law is one about an action's 'non-effects', i.e., about the facts not impacted by a specific action. They are called *frame axioms* and are needed when reasoning under an open-world assumption. In a propositional multi-modal language, they have the form $\ell \rightarrow \Box_a \ell$, where ℓ is a *literal*, i.e., $\ell = p$ or $\ell = \neg p$, for some $p \in \mathcal{P}$. An example of a frame axiom is $\mathsf{alive} \rightarrow \Box_{\mathsf{wait}} \mathsf{alive}$. Not surprisingly, some frame axioms may also fail: if the turkey is too old and about to die of natural causes, we are not guaranteed to find it alive after waiting. A *defeasible frame axiom* (DFA) is a statement of the form $\ell \mathrel{|\!\sim} \Box_a \ell$, where ℓ is a literal and $a \in \mathcal{A}$, and is read as "usually, the execution of action a does not

change the status of ℓ." For example, we could have $\mathsf{alive} \mathrel{|\!\sim} \Box_{\mathsf{wait}}\mathsf{alive}$, specifying that usually, we do not find a dead turkey after just waiting.

Executability and *inexecutability laws* make explicit, respectively, the known preconditions for an action to be executed and the circumstances preventing its execution. In a propositional multi-modal setting, executability laws take the form $\alpha \rightarrow \Diamond_a \top$, whereas inexecutability laws are of the form $\alpha \rightarrow \Box_a \bot$. Examples of each are, respectively, $\mathsf{loaded} \rightarrow \Diamond_{\mathsf{shoot}} \top$ and $\neg\mathsf{loaded} \rightarrow \Box_{\mathsf{shoot}} \bot$. (Note that, in a modal language, inexecutability laws can also be seen as a special case of effect laws in which the effect is $\bot$.)

Similarly to the previous types of laws we have seen, executability and inexecutability laws may fail. Indeed, in the (abnormal) situation in which the gun is loaded but the bullet is stuck in the barrel, one cannot shoot. Furthermore, the unusual situation of the Rust movie set,[1] in which someone was shot and killed with a technically unloaded gun, remains foreseeable.

A *defeasible executability law* (DXL) is a statement of the form $\alpha \mathrel{|\!\sim} \Diamond_a \top$, where α is a *propositional* sentence, and is read as "usually, if α holds, then a is executable." For instance, $\mathsf{loaded} \mathrel{|\!\sim} \Diamond_{\mathsf{shoot}} \top$ conveys the idea that in the normal situations where the gun is loaded, it is possible to shoot. This is in line with the intuitions and also caters for the less usual case we motivated above. Similarly, a *defeasible inexecutability law* (DIL) is a statement of the form $\alpha \mathrel{|\!\sim} \Box_a \bot$, with α a *propositional* sentence, and is read as "usually, α prevents a's execution." (Just as in the classical modal case, DILs can be seen as a special kind of DELs — see above — in which the expected outcome of an action, in a given situation, is always false.) As an example, $\neg\mathsf{loaded} \mathrel{|\!\sim} \Box_{\mathsf{shoot}} \bot$ specifies that, usually, one cannot shoot with an unloaded gun.

Notice that, since defeasible laws have a rule-like flavour, $\mathrel{|\!\sim}$ is *not* allowed to be nested in each type of defeasible law we have introduced above. This assumption is useful in showing a representation result w.r.t. the set of postulates characterising $\mathrel{|\!\sim}$'s behaviour (cf. Section 4).

A *defeasible action domain description*, denoted $\mathcal{KB}$ (for knowledge base), is a finite set of defeasible laws of the above-introduced types, possibly containing classical modal sentences.

[1] https://en.wikipedia.org/wiki/Rust_(2024_film)

Example 1. *The following is an example of a defeasible action domain description in the shooting scenario:* $\mathcal{KB} = \{\mathsf{walking} \mathrel{|\!\sim} \mathsf{alive},\ \neg\mathsf{loaded} \mathrel{|\!\sim} \Box_{\mathsf{load}}\mathsf{loaded},\ \mathsf{loaded} \mathrel{|\!\sim} \Box_{\mathsf{shoot}}\neg\mathsf{alive},\ \mathsf{hasGun} \mathrel{|\!\sim} \Diamond_{\mathsf{shoot}}\top,\ \neg\mathsf{hasGun} \mathrel{|\!\sim} \Box_{\mathsf{shoot}}\bot,\ \Diamond_{\mathsf{wait}}\top,\ \mathsf{alive} \mathrel{|\!\sim} \Box_{\mathsf{wait}}\mathsf{alive},\ \mathsf{loaded} \mathrel{|\!\sim} \Box_{\mathsf{wait}}\mathsf{loaded}\}$.

Intuitively, one expects defeasible domain descriptions to be more tolerant to exceptions regarding the behaviour of actions, i.e., to conflicting information, which leads to inconsistency when classical reasoning is assumed. In the next section, we see how defeasible laws can be given an intuitive semantics which, later on, will lend itself to a suitable notion of entailment from a defeasible action domain description.

4 Rational semantics

Defeasibility (or non-monotonicity) *tout court* is not enough: to be meaningful and useful, defeasible-reasoning processes need to be performed in a principled way. This amounts to satisfying a set of formal properties or postulates as they are usually referred to in the literature. Among these, rationality (and its various guises) is traditionally considered the baseline for reasoning about the real world. We now show how this requirement can be captured in the modal preferential semantics of Britz et al. [3, 4, 5].

Given a set X, the binary relation $\prec \subseteq X \times X$ is a *ranked order* if there is a mapping $r : X \longrightarrow \mathbb{N}$ satisfying the *convexity property* (for every $i \in \mathbb{N}$, if for some $x \in X$ $r(x) = i$, then, for every j s.t. $0 \leq j < i$, there is a $y \in X$ for which $r(y) = j$), and s.t. for every $x, y \in X$, $x \prec y$ if $r(x) < r(y)$. The idea is that $r(x)$ denotes the 'rank' of x in the set X, the reason $\prec$ induced by $r(\cdot)$ as above is called a ranked order.

Definition 1. *A* ***ranked Kripke model*** *is a tuple* $\mathscr{R} \stackrel{\text{def}}{=} \langle W, R, V, \prec\rangle$, *where* $\langle W, R, V\rangle$ *is a Kripke model and* $\prec$ *is a ranked order on* W.

It can be shown that, for every ranked Kripke model, the function $r(\cdot)$ is unique, i.e., given a ranked Kripke model $\mathscr{R} = \langle W, R, V, \prec\rangle$, there is only one function $r : W \longrightarrow \mathbb{N}$ satisfying the convexity property above and such that for every $w, u \in W$, $w \prec u$ iff $r(w) < r(u)$. (The proof is analogous to that by Britz et al. [2] in the description logic case.) This result allows us to talk about the *characteristic ranking* $r^{\mathscr{R}}(\cdot)$ associated

with a ranked Kripke model $\mathscr{R}$, which will be useful in the semantic constructions in Section 5.

Intuitively, the lower the rank of a world in a ranked Kripke model $\mathscr{R}$, the more typical (or normal) the world is in $\mathscr{R}$.

Figure 1 depicts an example of a ranked Kripke model for $\mathcal{P} = \{\mathsf{alive}, \mathsf{hasGun}, \mathsf{loaded}, \mathsf{walking}\}$ and $\mathcal{A} = \{\mathsf{entice}, \mathsf{load}, \mathsf{shoot}, \mathsf{wait}\}$.

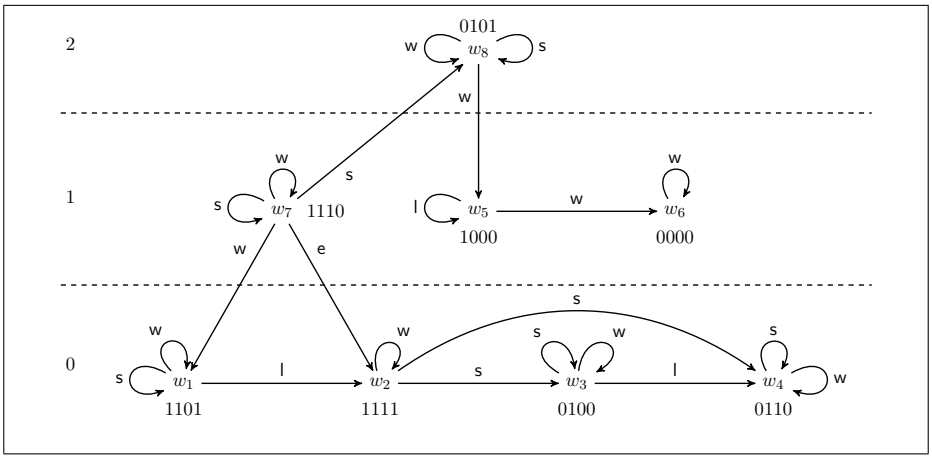

Figure 1: A ranked Kripke model for $\mathcal{P} = \{\mathsf{alive}, \mathsf{hasGun}, \mathsf{loaded}, \mathsf{walking}\}$ (with truth values featuring in this order in valuations) and $\mathcal{A} = \{\mathsf{entice}, \mathsf{load}, \mathsf{shoot}, \mathsf{wait}\}$ (names abbreviated for conciseness). Ranks are shown vertically on the left.

Given a ranked Kripke model $\mathscr{R}$ and $\alpha \in \mathcal{L}$, the definition of $[\![\alpha]\!]^{\mathscr{R}}$ is extended in the obvious way. Armed with ranked Kripke models, one can give a semantics to $\mathrel{|\!\sim}$-statements: $\mathscr{R} \Vdash \alpha \mathrel{|\!\sim} \beta$ if $\min_{\prec}[\![\alpha]\!]^{\mathscr{R}} \subseteq [\![\beta]\!]^{\mathscr{R}}$, i.e., the minimal α-worlds w.r.t. $\prec$ in $\mathscr{R}$ are β-worlds.

One of the consequences of our semantics is that for every ranked Kripke model $\mathscr{R} = \langle W, R, V, \prec \rangle$ and every $\alpha \in \mathcal{L}$, α is true in $\mathscr{R}$, i.e., $[\![\alpha]\!]^{\mathscr{R}} = W$ iff $\mathscr{R} \Vdash \neg\alpha \mathrel{|\!\sim} \bot$. Hence, every classical modal sentence α can be seen as just an abbreviation for the defeasible statement $\neg\alpha \mathrel{|\!\sim} \bot$.

We say a ranked Kripke model $\mathscr{R}$ *satisfies* (alias is a *model* of) an action domain description $\mathcal{KB}$ if $\mathscr{R}$ satisfies every statement in $\mathcal{KB}$. As an example, the ranked Kripke model depicted in Figure 1 satisfies the action domain description $\mathcal{KB}$ in Example 1.

Theorem 1 (Finite-Model Property)**.** *The logic of modal $\mathrel{|\!\sim}$-statements has the finite-model property: every defeasible action domain description*

that has a ranked Kripke model also has a finite ranked Kripke model, i.e., one in which the set W is finite.

In the literature on non-monotonic reasoning, there is an agreement that, in order to be considered *rational*, $\mathrel{|\!\sim}$ ought to satisfy all the properties shown in Figure 2, which have been put forward by Kraus, Lehmann and Magidor [16], and usually referred to as the KLM postulates:

$$\text{(Ref)}\quad \alpha \mathrel{|\!\sim} \alpha \qquad \text{(LLE)}\quad \frac{\models \alpha \leftrightarrow \beta,\ \alpha \mathrel{|\!\sim} \gamma}{\beta \mathrel{|\!\sim} \gamma}$$

$$\text{(And)}\quad \frac{\alpha \mathrel{|\!\sim} \beta,\ \alpha \mathrel{|\!\sim} \gamma}{\alpha \mathrel{|\!\sim} \beta \wedge \gamma} \qquad \text{(Or)}\quad \frac{\alpha \mathrel{|\!\sim} \gamma,\ \beta \mathrel{|\!\sim} \gamma}{\alpha \vee \beta \mathrel{|\!\sim} \gamma}$$

$$\text{(RW)}\quad \frac{\alpha \mathrel{|\!\sim} \beta,\ \models \beta \rightarrow \gamma}{\alpha \mathrel{|\!\sim} \gamma} \qquad \text{(CM)}\quad \frac{\alpha \mathrel{|\!\sim} \beta,\ \alpha \mathrel{|\!\sim} \gamma}{\alpha \wedge \beta \mathrel{|\!\sim} \gamma}$$

$$\text{(RM)}\quad \frac{\alpha \mathrel{|\!\sim} \beta,\quad \alpha \not\mathrel{|\!\sim} \neg\gamma}{\alpha \wedge \gamma \mathrel{|\!\sim} \beta}$$

Figure 2: KLM rationality properties or postulates.

(For more details on the postulates above, as well as on others, we refer the reader to the provided references [10, 16, 17].)

The following representation result, which is a reformulation of the one by Britz et al. for a pointed-model semantics [3, 4] and of which the proof follows that by Britz et al. in the defeasible description logic case [2], establishes the 'soundness' and 'completeness' of the KLM postulates above w.r.t. the class of ranked Kripke models:

Theorem 2. *Every ranked Kripke model $\mathscr{R}$ satisfies the KLM properties, i.e., whenever $\mathscr{R}$ satisfies the statements in the antecedent of a KLM property, it also satisfies the respective consequent. Conversely, if a set X of $\mathrel{|\!\sim}$-statements is rational, then there is a ranked Kripke model satisfying all and only the statements in X.*

5 Rationality in entailment

From the standpoint of knowledge representation and reasoning, a central question is determining which statements are entailed by a defeasible action domain description. Given the semantic constructions from the

previous section, the obvious starting point in the study of entailment in our setting is the following notion:

Definition 2 (Ranked Entailment)**.** *A statement $\alpha \mid\sim \beta$ is **rank entailed** by a defeasible action domain description $\mathcal{KB}$, denoted $\mathcal{KB} \models_{\mathsf{rk}} \alpha \mid\sim \beta$, if every ranked model of $\mathcal{KB}$ satisfies $\alpha \mid\sim \beta$.*

Let $\mathcal{KB}$ be a defeasible action domain description and let Δ be a fixed countably infinite set. With $Mod_{\Delta}(\mathcal{KB}) \stackrel{\text{def}}{=} \{\mathscr{R} = \langle W, R, V, \prec \rangle \mid \mathscr{R} \Vdash \mathcal{KB}, \mathscr{R}$ is ranked, and $W = \Delta\}$, we denote the set of Δ-*models of* $\mathcal{KB}$. It turns out that ranked entailment above can be fully characterised by the ranked Kripke models in $Mod_{\Delta}(\mathcal{KB})$, as the following modal version of a result by Britz et al. [2] establishes:

Lemma 1. *For every $\mathcal{KB}$ and every $\alpha, \beta \in \mathcal{L}$, $\mathcal{KB} \models_{\mathsf{rk}} \alpha \mid\sim \beta$ iff $\mathscr{R} \Vdash \alpha \mid\sim \beta$, for every $\mathscr{R} \in Mod_{\Delta}(\mathcal{KB})$.*

Nevertheless, as already shown by Britz et al. [3], ranked entailment is not satisfactory in a non-monotonic setting, the crux of the matter being it remains a Tarskian notion of entailment and, hence, is monotonic. This is similar to well-known results in the propositional [17] and description logic [2] cases. The next example captures the essence of the argument against ranked entailment: it is neither *ampliative* nor *defeasible*, thereby failing to preserve rational monotonicity (RM) in Figure 2.

Example 2. *Let us assume the simple defeasible action domain description $\mathcal{KB} = \{\mathsf{walking} \rightarrow \mathsf{alive}, \mathsf{alive} \mid\sim \Diamond_{\mathsf{entice}} \top\}$, specifying that a walking turkey is known for sure to be alive, and that a live turkey can usually be enticed. It can be checked that $\mathcal{KB} \not\models_{\mathsf{rk}} \mathsf{walking} \mid\sim \Diamond_{\mathsf{entice}} \top$, i.e., ranked entailment does not allow us to draw the (plausible) conclusion that walking turkeys can usually (at least provisionally) be enticed. The only way to ensure this conclusion is by adding it explicitly to $\mathcal{KB}$, getting $\mathcal{KB}' = \mathcal{KB} \cup \{\mathsf{walking} \mid\sim \Diamond_{\mathsf{entice}} \top\}$, which gets us into trouble if we ever learn that (for whatever reason) a walking turkey cannot be enticed: $\mathcal{KB}'' = \mathcal{KB}' \cup \{\mathsf{walking} \mid\sim \Box_{\mathsf{entice}} \bot\} \models_{\mathsf{rk}} \neg\mathsf{walking}$, i.e., turkeys never walk, which is an unintuitive conclusion in our scenario.*

To ensure rationality when reasoning with defeasible action domain descriptions, we need to move beyond the (monotonic) notion of consequence that Definition 2 embodies. The literature on non-monotonic

reasoning offers us valuable insights in this direction. The constructions we present now are inspired by the semantic characterisation of rational closure by Booth and Paris in the propositional case [1] and are based mainly on its extension to description logics by Britz et al. [2].

Given a set of ranked Kripke models, one can merge them by extending a standard operation of the classical modal semantics.

Definition 3 (Ranked Union)**.** *Given a countable set of ranked Kripke models $\mathcal{R} = \{\mathscr{R}_1, \mathscr{R}_2, \ldots\}$, with $\mathscr{R}^{\mathcal{R}} \stackrel{\text{def}}{=} \langle W^{\mathcal{R}}, R^{\mathcal{R}}, V^{\mathcal{R}}, \prec^{\mathcal{R}} \rangle$ we denote the **ranked union** of $\mathcal{R}$, where:*

- *$W^{\mathcal{R}} \stackrel{\text{def}}{=} \coprod_{\mathscr{R} \in \mathcal{R}} W$, i.e., the disjoint union of the worlds from $\mathcal{R}$, where each $\mathscr{R} \in \mathcal{R}$ has the elements $w, u, \ldots$ of its W renamed as $w_{\mathscr{R}}$, $u_{\mathscr{R}}$, ... so that they are all distinct in $W^{\mathcal{R}}$;*
- *$V^{\mathcal{R}}(w_{\mathscr{R}}) = V(w)$ in $\mathscr{R}$, and therefore $\mathscr{R}^{\mathcal{R}}, w_{\mathscr{R}} \Vdash p$ iff $\mathscr{R}, w \Vdash p$;*
- *$(w_{\mathscr{R}}, w'_{\mathscr{R}'}) \in R_a^{\mathcal{R}}$ iff $\mathscr{R} = \mathscr{R}'$ and $(w, w') \in R_a$ in $\mathscr{R}$;*
- *for every $w_{\mathscr{R}} \in W^{\mathcal{R}}$, $r^{\mathscr{R}^{\mathcal{R}}}(w_{\mathscr{R}}) = r^{\mathscr{R}}(w)$, i.e., renamed worlds keep their ranks from the respective $\mathscr{R}$ (cf. Definition 1 and below it).*

The latter condition corresponds to imposing that $w_{\mathscr{R}} \prec^{\mathcal{R}} w'_{\mathscr{R}'}$ if and only if $r^{\mathscr{R}}(w) < r^{\mathscr{R}'}(w')$.

Informally, the ranked union of a set of ranked Kripke models is the result of merging all their ranks of value i into a single rank of value i, for each i. It can be shown that the ranked union built up from a set of ranked Kripke models of a knowledge base $\mathcal{KB}$ is itself a ranked Kripke model of $\mathcal{KB}$. (The proof is similar to that of an analogous result in the description logic case [2, Lemma 8].)

Using the definitions of $Mod_{\Delta}(\mathcal{KB})$ (see previous page) and of ranked union, we can construct a canonical ranked Kripke model of $\mathcal{KB}$.

Definition 4 (Big Ranked Kripke Model)**.** *Let $\mathcal{KB}$ be a defeasible action domain description. The **big ranked Kripke model** of $\mathcal{KB}$ is the ranked Kripke model $\mathscr{R}^{\mathcal{R}}$ such that $\mathcal{R} = Mod_{\Delta}(\mathcal{KB})$.*

Definition 5 (Rational Entailment)**.** *A statement $\alpha \mathrel{|\!\sim} \beta$ is **rationally entailed** by a defeasible action domain description $\mathcal{KB}$, denoted $\mathcal{KB} \mathrel{|\!\approx}_{\mathsf{rat}} \alpha \mathrel{|\!\sim} \beta$, if $\mathscr{R}^{\mathcal{R}} \Vdash \alpha \mathrel{|\!\sim} \beta$, where $\mathscr{R}^{\mathcal{R}}$ is the big ranked Kripke model of $\mathcal{KB}$.*

The following result establishes that rational entailment is a suitable notion of semantic entailment in our setting.

Proposition 1. $\{\alpha \mid\sim \beta \mid \mathcal{KB} \mid\approx_{\mathsf{rat}} \alpha \mid\sim \beta\}$ *is rational, i.e., satisfies all the properties in Figure 2.*

Example 3. *Coming back to Example 2, it can be shown that* $\mathcal{KB} \mid\approx_{\mathsf{rat}} \mathsf{walking} \mid\sim \Diamond_{\mathsf{entice}}\top$. *Furthermore, if* $\mathcal{KB}' = \mathcal{KB} \cup \{\mathsf{walking} \mid\sim \Box_{\mathsf{entice}}\bot\}$, *then* $\mathsf{walking} \mid\sim \Diamond_{\mathsf{entice}}\top$ *is no longer sanctioned, and* $\mathcal{KB}' \not\mid\approx_{\mathsf{rat}} \neg\mathsf{walking}$, *which is in line with the intuitions.*

Of course, to reason rationally with a defeasible action domain description, one needs a procedure capable of deciding rational entailment. It turns out the algorithm for computing the rational closure of a knowledge base by Britz et al. [2] can easily be adapted to the modal language we have assumed here, thereby giving us a decision procedure for checking rational entailment from defeasible action domain descriptions. (Space considerations prevent us from providing the details here.)

6 Discussion and open questions

The following discussion assumes the reader's acquaintance with the area of reasoning about actions and with some of Andi's work.

6.1 The frame and ramification problems

An obvious question to ask now is how the rational framework thus defined stands w.r.t. two of the historically most challenging problems in reasoning about actions. In what follows, we assume the defeasible action domain description $\mathcal{KB}$ from Example 1.

Concerning the frame problem, it can be verified that $\mathcal{KB} \not\mid\approx_{\mathsf{rat}} \mathsf{loaded} \rightarrow \Box_{\mathsf{entice}}\mathsf{loaded}$, i.e., in the big ranked model of $\mathcal{KB}$, the (classical) frame axiom $\mathsf{loaded} \rightarrow \Box_{\mathsf{entice}}\mathsf{loaded}$ is not true. This means there are situations resulting from enticing the turkey in which the gun gets unloaded. This is because worlds satisfying $\mathsf{loaded} \wedge \Diamond_{\mathsf{entice}}\neg\mathsf{loaded}$ do not get removed by the disjoint union operation, obviously.

One would then expect the defeasible version of such a frame axiom, namely $\mathsf{loaded} \mid\sim \Box_{\mathsf{entice}}\mathsf{loaded}$, to always hold. As it turns out, we get

$\mathcal{KB} \not\mid\!\sim_{\mathsf{rat}} \mathsf{loaded} \mid\!\sim \Box_{\mathsf{entice}}\mathsf{loaded}$, too. This is perhaps less obvious to see than the classical case above, but the argument is roughly as follows: in the construction of the big ranked Kripke model of $\mathcal{KB}$ above, nothing prevents us from having a ranked Kripke model $\mathscr{R}$ in $Mod_{\Delta}(\mathcal{KB})$ in which there is a possible world w s.t. $w \in \min_{\prec}[\![\mathsf{loaded}]\!]^{\mathscr{R}}$ and $\mathscr{R}, w \Vdash \Diamond_{\mathsf{entice}}\neg\mathsf{loaded}$. Notice this does not happen regarding wait and loaded since the DFA $\mathsf{loaded} \mid\!\sim \Box_{\mathsf{wait}}\mathsf{loaded}$ is *explicitly* stated in $\mathcal{KB}$.

As a result, and, in retrospect, not surprisingly, rationality alone is not enough to ensure that the relevant frame axioms hold without stating them explicitly in the knowledge base.

Moving now to the ramification problem, one can see that, given $\mathcal{KB}$ from Example 1, $\mathcal{KB} \not\mid\!\sim_{\mathsf{rat}} \mathsf{loaded} \rightarrow \Box_{\mathsf{shoot}}\neg\mathsf{walking}$. Even the defeasible ramification, i.e., $\mathsf{loaded} \mid\!\sim \Box_{\mathsf{shoot}}\neg\mathsf{walking}$, is not warranted by $\mathcal{KB}$. The reason is as follows: (i) $\neg\mathsf{alive} \mid\!\sim \neg\mathsf{walking}$ does not follow from $\mathsf{walking} \mid\!\sim \mathsf{alive}$, given $\mid\!\sim$'s properties, and (ii) even if $\neg\mathsf{alive} \mid\!\sim \neg\mathsf{walking}$ is explicitly enforced in $\mathcal{KB}$, not all possible executions of shoot land at a *most normal* $\neg\mathsf{alive}$-situation, and therefore $\neg\mathsf{walking}$ is not always ensured. Hence, the so-called 'indirect' effects must be explicitly stated.

The bottom line is that our rational modal framework needs to be equipped with a causality-based solution to the frame and ramification problems. Andi's work on dependence relations [6, 7], which provides an elegant solution to both the frame and ramification problems in the classical case, can naturally be adapted to achieve that in our defeasible setting. (We shall omit the details due to space considerations.)

6.2 Rationality and regression

Reiter [18] has shown that in scenarios with only deterministic actions and no ramifications, a simple solution to the frame problem is possible. Roughly, it amounts to compiling effect laws and explanation closure axioms [18] into successor-state axioms (SSAs), which give the necessary and sufficient conditions for propositions to hold (or not) after an action's execution. Moreover, Reiter has shown that SSAs can be used to reduce entailment checking in a first-order action formalism to propositional satisfiability, through a rewriting procedure called *regression*.

Andi has had the insight of recasting regression in a modal setting [9], which has proven fruitful beyond reasoning about actions, viz. in epis-

temic reasoning, thereby strengthening the case of modal logics as a viable alternative to the situation calculus.

The move to a rational multi-modal logic of actions as the one we consider here raises the question of how a suitable version of regression *à la* Andi in this setting can be defined. In particular, a solution to the frame problem allowing for rational entailment to be reduced to rational closure in the propositional case would be a useful result. It turns out this is not as straightforward as it might seem at first sight. Below, we point out some of the difficulties brought about by the properties of $\mid\!\sim$ and sketch a potential workaround in a more restricted case.

In our shooting scenario, an example of a classical SSA would be $\Box_{\mathsf{shoot}}\neg\mathsf{alive} \leftrightarrow (\neg\mathsf{hasGun} \vee \mathsf{loaded} \vee \neg\mathsf{alive})$. This enables us to replace every occurrence of $\Box_{\mathsf{shoot}}\neg\mathsf{alive}$ in a complex query with $\neg\mathsf{hasGun} \vee \mathsf{loaded} \vee \neg\mathsf{alive}$, thereby decreasing the modal depth of the query of one. Successive applications of this principle to other modal subsentences, along with some normalisation rules holding in the deterministic case, eventually lead to a classical propositional sentence, of which the validity can be checked by a state-of-the-art SAT solver.

Obviously, for regression to be applicable to a query containing $\Box_a\alpha$ as a subsentence, one needs a suitable form of equivalence, either at the object level (in the form of a biconditional) or at the meta-level. This amounts to using either classical equivalence or some yet-to-be-defined form of 'defeasible equivalence' allowing for substitution of $\Box_a\alpha$ by the corresponding equivalent sentence. The latter case remains, to the best of our knowledge, an open question in the NMR literature. The former means we allow only classical sentences in the knowledge base (or assume a Tarskian-style logical equivalence at the meta-level).

In general, one cannot generate classical SSAs from defeasible laws without losing their defeasible behaviour, which is the purpose of extending the modal language with $\mid\!\sim$ in the first place. This raises a few questions, among which are "What are defeasible SSAs?", "What are the implications of reasoning in their presence?", "Does that limit regression?" These are questions that we shall for now leave open.

Under the assumption that we allow DICs but only classical action laws, and assuming deterministic actions, without ramifications, one can compile SSAs as in the classical modal case and apply regression. It is

still possible for queries to be defeasible modal statements of the form $\alpha \mathrel{|\!\sim} \Box_{a_1} \cdots \Box_{a_n} \beta$, which adds to the expressive power of classical action domain descriptions and their reasoning services. In this case, the defeasible query is reducible, via an Andi-style regression, to a propositional defeasible conditional of the form $\alpha \mathrel{|\!\sim} \gamma$, where γ is a propositional sentence and of which the validity can then be checked through the rational closure algorithm for propositional logic.

6.3 Unwanted implicit laws

Classical as well as non-classical knowledge bases often entail unwanted or unexpected conclusions. These may be due to logical inconsistency, but also show up as a result of poor design in the domain specification. To witness, in a classical modal setting, from $\mathsf{hasGun} \rightarrow \Diamond_{\mathsf{shoot}} \top$ and $\neg\mathsf{loaded} \rightarrow \Box_{\mathsf{shoot}} \bot$ we conclude $\mathsf{hasGun} \rightarrow \mathsf{loaded}$, i.e., it is impossible to have an unloaded gun. The latter is an instance of an *implicit integrity constraint*. Other types of (unwanted as well as wanted) implicit laws have also been studied by Andi and colleagues [12, 15]. In particular, a notion of modularity [13, 14] has been put forward as an approach to making sure knowledge engineers can detect implicit consequences more easily and also repair the domain description if needed [19].

The shift to defeasible action domain descriptions under rationality offers a promising answer to the issue of unwanted implicit laws. Indeed, for the case of integrity constraints, from $\alpha \mathrel{|\!\sim} \Diamond_a \top$ and $\beta \mathrel{|\!\sim} \Box_a \bot$ it does *not* follow that $\alpha \wedge \beta \mathrel{|\!\sim} \bot$ must hold.

Acknowledgments

I want to thank the organisers of this volume for the opportunity to pay homage to someone who has been much more than a PhD advisor and whose influence on my career has gone beyond my doctorate. Thanks to the two anonymous referees for their useful comments and suggestions.

References

[1] R. Booth and J.B. Paris. A note on the rational closure of knowledge bases with both positive and negative knowledge. *JoLLI*, 7(2):165–190, 1998.

[2] K. Britz, G. Casini, T. Meyer, K. Moodley, U. Sattler, and I. Varzinczak. Principles of KLM-style defeasible description logics. *ACM Transactions on Computational Logic*, 22(1):1–46, 2021.

[3] K. Britz, T. Meyer, and I. Varzinczak. Preferential reasoning for modal logics. In *Proc. of M4M*, pages 55–69, 2011.

[4] K. Britz, T. Meyer, and I. Varzinczak. Normal modal preferential consequence. In *Australasian Joint Conf. on AI*, pages 505–516, 2012.

[5] K. Britz and I. Varzinczak. From KLM-style conditionals to defeasible modalities, and back. *JANCL*, 28(1):92–121, 2018.

[6] M.A. Castilho, O. Gasquet, and A. Herzig. Formalizing action and change in modal logic I: the frame problem. *JLC*, 9(5):701–735, 1999.

[7] M.A. Castilho, A. Herzig, and I. Varzinczak. It depends on the context! A decidable logic of actions and plans based on a ternary dependence relation. In *Proc. of NMR*, 2002.

[8] B. Chellas. *Modal logic: An introduction*. Cambridge Univ. Press, 1980.

[9] R. Demolombe, A. Herzig, and I. Varzinczak. Regression in modal logic. *JANCL*, 13(2):165–185, 2003.

[10] N. Friedman and J.Y. Halpern. Plausibility measures and default reasoning. *Journal of the ACM*, 48(4):648–685, 2001.

[11] D. Harel, J. Tiuryn, and D. Kozen. *Dynamic Logic*. MIT Press, 2000.

[12] A. Herzig and I. Varzinczak. Domain descriptions should be modular. In *Proc. of ECAI*, pages 348–352. IOS Press, 2004.

[13] A. Herzig and I. Varzinczak. On the modularity of theories. In *Advances in Modal Logic*, 5, pages 93–109. King's College Publications, 2005.

[14] A. Herzig and I. Varzinczak. A modularity approach for a fragment of $\mathcal{ALC}$. In *Proc. of JELIA*, pages 216–228. Springer-Verlag, 2006.

[15] A. Herzig and I. Varzinczak. Metatheory of actions: beyond consistency. *Artificial Intelligence*, 171:951–984, 2007.

[16] S. Kraus, D. Lehmann, and M. Magidor. Nonmonotonic reasoning, preferential models and cumulative logics. *Artif. Intelligence*, 44:167–207, 1990.

[17] D. Lehmann and M. Magidor. What does a conditional knowledge base entail? *Artificial Intelligence*, 55:1–60, 1992.

[18] R. Reiter. *Knowledge in Action: Logical Foundations for Specifying and Implementing Dynamical Systems*. MIT Press, 2001.

[19] I.J. Varzinczak. On action theory change. *JAIR*, 37:189–246, 2010.