Contracting TBoxes: the importance of being modular

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Abstract

In this work we address the problem of contracting terminological axioms in description logics. We present the semantics of ontology change for a fragment of $\mathcal{ALC}$, and also define the corresponding syntactical operators for contracting TBoxes. We then take profit of a concept of modularity of TBoxes in order to establish soundness and completeness results for modular ontologies.

1 Introduction

Suppose we have an agent designed for an automatic passport control system in an airport. The agent's knowledge base might be made up of an ontology containing a terminology about passengers, as well as assertions describing specific states of affairs of such an environment.

Suppose that the agent's terminology states that "a foreigner is someone from outside the European Union". This can be encoded in description logics like $\mathcal{ALC}$ by the axiom $\mathsf{Foreigner} \equiv \neg\mathsf{EUcitizen}$. Suppose now that one day the agent gets the information that there is a Brazilian passenger who is also an EU citizen. In such a case, the agent must change her beliefs about the relationship between concepts $\mathsf{Foreigner}$ and $\mathsf{EUcitizen}$: there can be people with double citizenship. This example is an instance of the problem of changing propositional belief bases and is largely addressed in the literature about belief change [6] and belief update [14].

Next, suppose our agent's terminology contain the statement $\mathsf{EUcitizen} \equiv \forall\mathsf{passport.EU}$, i.e., "EU citizens have only EU passports". This means that if someone is an EU citizen, all her passports must have been delivered by some EU member country. Now suppose a person with double citizenship arrives at the passport control desk and, despite having EU citizenship, she surprisingly shows a Brazilian passport. This means that $\mathsf{EUcitizen} \equiv \forall\mathsf{passport.EU}$ has to be given up.

Imagine now that the agent believes that "every passenger possesses a passport": $\mathsf{Passenger} \sqsubseteq \exists\mathsf{passport}.\top$. This means that, in order to be a valid passenger someone must have a passport. However, one day there is someone without a passport claiming that she can travel anyway by just showing her ID card. If this gets accepted, then $\mathsf{Passenger} \sqsubseteq \exists\mathsf{passport}.\top$ has to be retracted from the terminology. Alternatively, the agent's terminology could have the information that in order to get into Czech Republic, a Brazilian passenger must have a visa. But at some point the Czech Republic joined the European Union, and hence Brazilian tourists no longer need a visa to taste a beer in Kafka's country.

The examples above illustrate situations where terminological axioms must be changed. In the first example, being a non-EU citizen is shown not to be a complete definition of a foreigner, the latter actually being a specialization of the former. In the second example, having only EU passports, once believed to define an EU citizen, has now to be seen as a definition for those EU citizens who do not have double citizenship. In the last examples, the need for an attribute under concern is questioned in the light of new information showing a context where it is no longer mandatory. Carrying out modifications like those is what we here call *contracting* a terminology.

In the DL literature, it seems to be more or less tacitly assumed that TBoxes are designed once for all and are not likely to evolve, while ABoxes are more likely to change. However, our examples show that TBoxes may also change.

Up until now, theory change has been studied mainly for knowledge bases in classical logic, both in terms of revision and update. Only in a few recent works it has been considered in the realm of modal logics, viz. in epistemic logic [8], in action languages [4] and in PDL [9]. Recently, several works [18, 12] have investigated revision of beliefs about facts of the world, which (in description logic terms) corresponds to revising ABoxes. In our examples, this would concern e.g. the current status of the passenger: the agent believes he is not an EU citizen, but is wrong about this and might subsequently be forced to revise her beliefs about the current state of affairs. Such belief revision operations do not modify the agent's terminology. In opposition to that, here we are interested exactly in such modifications. The aim of this work is to make a step toward that issue and propose a framework that deals with the contraction of terminological axioms.

2 Background

2.1 Logical preliminaries

In $\mathcal{ALC}$ we use A to denote atomic concepts, R for atomic roles from a set $\mathfrak{Roles} = \{R_1, R_2, \ldots\}$, and $C, D, \ldots$ for complex concept descriptions. These are recursively defined in the following way:

$$C ::= A \mid \top \mid \bot \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \forall R.C \mid \exists R.C$$

In our example, $\mathsf{Foreigner} \sqcap \mathsf{EUcitizen}$, $\forall \mathsf{passport}.\mathsf{EU}$ and $\exists \mathsf{passport}.\top$ are complex concepts in $\mathcal{ALC}$.

Definition 1 *An* interpretation $\mathcal{I}$ *is a tuple* $\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ *such that* $\Delta^{\mathcal{I}}$ *is a nonempty set and* $\cdot^{\mathcal{I}}$ *a function mapping:*

- *every concept to a subset of* $\Delta^{\mathcal{I}}$*;*
- *every role to a subset of* $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$*.*

Given $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, $\Delta^{\mathcal{I}}$ is the *domain*, and $\cdot^{\mathcal{I}}$ the associated *interpretation function*. If a is an individual, A an atomic concept, R an atomic role, and C, D concepts, we have:

$$\begin{gathered} A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}},\ R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}, \\ \top^{\mathcal{I}} = \Delta^{\mathcal{I}},\ \bot^{\mathcal{I}} = \emptyset,\ (\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}, \\ (C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}},\ (C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}, \\ (\forall R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \ :\ \forall b.(a,b) \in R^{\mathcal{I}} \text{ implies } b \in C^{\mathcal{I}}\}, \\ (\exists R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \ :\ \exists b.(a,b) \in R^{\mathcal{I}} \text{ and } b \in C^{\mathcal{I}}\} \end{gathered}$$

Concept inclusion axioms (alias *axioms* or *subsumptions*) are of the form $C \sqsubseteq D$. *Concept definitions* $C \equiv D$ abbreviate $C \sqsubseteq D$ and $D \sqsubseteq C$.

Intuitively, $C \equiv D$ gives a definition for concept C in terms of D. In our example, $\mathsf{DoubleCitizen} \equiv \mathsf{Foreigner} \sqcap \mathsf{EUcitizen}$ gives necessary and sufficient conditions for a person to have double citizenship.

An interpretation $\mathcal{I}$ satisfies a subsumption $C \sqsubseteq D$ (noted $\mathcal{I} \models C \sqsubseteq D$) if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$. Intuitively, $C \sqsubseteq D$ means that C is more specific than D. In our example, $\mathsf{DoubleCitizen} \sqsubseteq \mathsf{EUcitizen}$ says that someone holding two citizenships is a specialization of a European citizen; and $\mathsf{Passenger} \sqsubseteq \exists \mathsf{passport}.\top$ says that a necessary condition to be a passenger is having a passport.

We call a set of axioms $\mathcal{T}$ a *terminology*, alias TBox. An interpretation $\mathcal{I}$ is a *model* of $\mathcal{T}$ (noted $\mathcal{I} \models \mathcal{T}$) if $\mathcal{I} \models C \sqsubseteq D$ for all $C \sqsubseteq D \in \mathcal{T}$. An axiom $C \sqsubseteq D$ is a *consequence* of $\mathcal{T}$ (noted $\mathcal{T} \models C \sqsubseteq D$) if for every $\mathcal{I}$, $\mathcal{I} \models \mathcal{T}$ implies $\mathcal{I} \models C \sqsubseteq D$.

2.2 Simple inclusion axioms

Let C be a concept. The mapping $\mathit{roles}(C)$ returns the set of roles occurring in C. For instance $\mathit{roles}(\exists R_1.D \sqcap \forall R_2.E) = \{R_1, R_2\}$. If $\mathit{roles}(C) = \emptyset$, we call C a *boolean concept*. For an axiom $C \sqsubseteq D$, $\mathit{roles}(C \sqsubseteq D) = \mathit{roles}(C) \cup \mathit{roles}(D)$. Moreover, for a TBox $\mathcal{T}$, let $\mathit{roles}(\mathcal{T}) = \bigcup_{C \sqsubseteq D \in \mathcal{T}} \mathit{roles}(C \sqsubseteq D)$.

We define two important kinds of axioms.

Definition 2 *An axiom $C \sqsubseteq D$ is a* boolean axiom *if $\mathit{roles}(C \sqsubseteq D) = \emptyset$. Else $C \sqsubseteq D$ is a* non-boolean axiom*.*

It will be useful to split $\cdot^{\mathcal{I}}$ into the interpretation of concepts and the interpretation of roles. We note the former mapping $\cdot^{\mathcal{I}}_{\kappa}$ and the latter $\cdot^{\mathcal{I}}_{\rho}$. $\mathcal{I}_{\kappa} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}_{\kappa} \rangle$ will be called a *boolean interpretation*. For terminologies $\mathcal{T}$ such that $\mathit{roles}(\mathcal{T}) = \emptyset$, boolean interpretations suffice.

Boolean interpretations will be represented alternatively as sets of valuations, where a valuation is a subset of the set of atomic concepts. Given a valuation v we can then construct a boolean interpretation $\mathcal{I}^{v}_{\kappa} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}_{\kappa} \rangle$ such that $\Delta^{\mathcal{I}} = \{v\}$ and $A^{\mathcal{I}}_{\kappa} = \{v \ : \ A \in v\}$. Then, the *canonical model* of a set of boolean axioms $\kappa\text{-}\mathit{model}(\mathcal{T}) = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}_{\kappa} \rangle$ where $\Delta^{\mathcal{I}} = \{v \ : \ v \text{ valuation , and } \mathcal{I}^{v}_{\kappa} \models \mathcal{T}\}$, and $A^{\mathcal{I}}_{\kappa} = \{v \ : \ A \in v\}$.

We also make a syntactical restriction on the non-boolean axioms in our TBoxes.

Definition 3 *If C is a boolean concept, then $\forall R.C$ is a* boolean value restriction*, and $\exists R.C$ is a* boolean existential restriction*.*

In our example, $\forall \mathsf{passport}.\mathsf{EU}$ and $\exists \mathsf{refund}.\mathsf{VAT}$ are, respectively, a value and an existential restriction.

Definition 4 *A* simple axiom *is a boolean axiom or a non-boolean axiom $C \sqsubseteq D$ such that $\mathit{roles}(C) = \emptyset$ and D is a boolean value or existential restriction.*

In our running example, $\mathsf{DoubleCitizen} \sqsubseteq \mathsf{Foreigner} \sqcap \mathsf{EUcitizen}$, $\mathsf{EUcitizen} \sqsubseteq \forall \mathsf{passport}.\mathsf{EU}$ and $\mathsf{Passenger} \sqsubseteq \exists \mathsf{passport}.\top$ are simple axioms.

Henceforth we suppose that:

All axioms in a TBox are simple axioms. (H)

Our fragment differs from $\mathcal{ALC}$ just in the sense that only boolean concepts are allowed in the scope of a quantification over a role. We observe however that we could allow for axioms with nested roles like $C \equiv \forall R_1.\forall R_2.D$ and GCIs like $\forall R_3.E \equiv \forall R_4.F$. For that it suffices to adapt an existing technique of *subformula renaming* for classical logic [17, 1] to recursively replace complex concepts with new concept names, stating definitions for these as global axioms. For instance, $C \equiv \forall R_1.\forall R_2.D$ should then be rewritten as $C \equiv \forall R_1.C'$ and $C' \equiv \forall R_2.D$, and $\forall R_3.E \equiv \forall R_4.F$ could be replaced by $E' \equiv \forall R_4.F$ and $E' \equiv \forall R_4.E$, with C', E' new concept names. It is known that renaming is satisfiability preserving and can be computed in polynomial time [16]. However it remains to assess the impact the introduction of new concept names can have on the intuition about the original ontology.

3 Semantics of contraction

When a knowledge base has to be changed, the basic operation is that of *contraction*. (In belief-base update [20, 14] it has also been called erasure.)

In this section we define contraction of an interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ by a concept inclusion axiom $C \sqsubseteq D$. The result will be a *set* of interpretations.

In general we might contract by any axiom $C \sqsubseteq D$. Here we focus on contraction by simple axioms. We therefore suppose that we contract by either $C \sqsubseteq D$, or $C \sqsubseteq \forall R.D$, or $C \sqsubseteq \exists R.D$, where $roles(C) = roles(D) = \emptyset$.

For the case of contracting by a boolean axiom, we suppose that it suffices to contract the set of boolean axioms of the TBox. To do that, we resort to existing approaches in order to change them. In the following, we can consider the DL counterpart of some belief change operator such as Forbus' update method [5], or the possible models approach [20, 21], or WSS [10] or MPMA [3].

Contraction by $C \sqsubseteq D$ means adding new objects to $\Delta^{\mathcal{I}}$. Let $\ominus$ be a contraction operator for classical logic.

Definition 5 *The set of models resulting from contracting an interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ by a boolean axiom $C \sqsubseteq D$ is the singleton $\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle^{-}_{C \sqsubseteq D} = \{\langle \Delta^{\mathcal{J}}, \cdot^{\mathcal{J}} \rangle\}$ such that $\mathcal{J}_\kappa = \mathcal{I}_\kappa \ominus \kappa\text{-}model(C \sqsubseteq D)$.*

One might object that some $R^{\mathcal{I}}$ might have to change as well, otherwise contracting a boolean axiom may conflict with non-boolean ones. For instance, if $C \sqsubseteq \exists R.\top \in \mathcal{T}$ and we contract by $C \sqsubseteq \bot$, the result may make $C \sqsubseteq \exists R.\top$ untrue. However, given the amount of information we have, we think that whatever we do with $R^{\mathcal{I}}$, we will always be able to find a counterexample to the intuitiveness of the operation, as it is domain dependent. For example, augmenting $R^{\mathcal{I}}$ may make a concept a specialization of another: if we have $C \equiv \forall R.D$ and we augment $R^{\mathcal{I}}$ such that $C \sqsubseteq \exists R.E$ is true in the resulting interpretation, we may loose the definition of C. Hence, deciding on what changes to carry out on $R^{\mathcal{I}}$ when contracting boolean axioms depends on the user's intuition, and this information cannot be generalized and established once for all. We here opt for a priori doing nothing with $R^{\mathcal{I}}$ and postponing correction of the other axioms in $\mathcal{T}$.

Now we consider contraction of non-boolean axioms. Suppose the knowledge engineer acquires new information regarding the value restrictions with role R. This means that the axioms with value restrictions mentioning R are probably too strong, i.e., there can be unforeseen special instances of a concept they describe, and thus they have to be weakened. Consider e.g. the axiom $\mathsf{EUcitizen} \sqsubseteq \forall \mathsf{passport}.\mathsf{EU}$, and suppose it has to be weakened to the more specific one $\mathsf{EUcitizen} \sqcap \neg\mathsf{DoubleCitizen} \sqsubseteq \forall \mathsf{passport}.\mathsf{EU}$, or, equivalently, $\mathsf{EUcitizen} \sqsubseteq (\mathsf{DoubleCitizen} \sqcup \forall \mathsf{passport}.\mathsf{EU})$.[1] In order to carry out such a weakening, first the designer has to contract the non-boolean axioms and second to expand the resulting set with the weakened axioms.

Contraction by $C \sqsubseteq \forall R.D$ amounts to changing $R^{\mathcal{I}}$ by adding any 'counterexample' links from $C^{\mathcal{I}}$ to $(\neg D)^{\mathcal{I}}$.

Definition 6 *The set of models resulting from contracting an interpretation* $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ *by an axiom* $C \sqsubseteq \forall R.D$ *is* $\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle^{-}_{C \sqsubseteq \forall R.D} = \{\langle \Delta^{\mathcal{J}}, \cdot^{\mathcal{J}} \rangle \ : \ \Delta^{\mathcal{J}} = \Delta^{\mathcal{I}}, \cdot^{\mathcal{J}} = \cdot^{\mathcal{I}}$ *except that* $R^{\mathcal{J}} = R^{\mathcal{I}} \cup \mathcal{R}$ *for some* $\mathcal{R} \subseteq C^{\mathcal{I}} \times \Delta^{\mathcal{I}}\}$.

Suppose now the knowledge engineer learns new information about the existential restrictions concerning the role R. This usually occurs when the non-boolean axioms with existential restrictions are too strong, i.e., the concept that is subsumed by the restriction under concern is too weak and has to be made more expressive. Let $\mathsf{Brazilian} \sqsubseteq \exists \mathsf{visa}.\mathsf{EU}$ be the law to be contracted, and suppose it has to be weakened to the more specific

[1]The other possibility of weakening the axiom, i.e., replacing it by $\mathsf{EUcitizen} \sqsubseteq \forall \mathsf{passport}.(\mathsf{EU} \sqcup \neg\mathsf{EU})$ looks silly. So we preferred to weaken the axiom by strengthening its left hand side.

$\mathsf{Brazilian} \sqcap \neg\mathsf{EUcitizen} \sqsubseteq \exists\mathsf{visa}.\mathsf{EU}$. To implement such a weakening, the designer has to first contract the set of axioms with existential restrictions and then to expand the result with the weakened axiom.

Contraction by $C \sqsubseteq \exists R.D$ corresponds to removing some R-links between C-individuals and D-individuals. Deletion of such pairs has as consequence that objects having property C are no longer required to be related with some object having property D by the relation under concern.

Definition 7 *The set of models that result from contracting an interpretation* $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ *by an axiom* $C \sqsubseteq \exists R.D$ *is* $\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle^{-}_{C \sqsubseteq \exists R.D} = \{\langle \Delta^{\mathcal{J}}, \cdot^{\mathcal{J}} \rangle \ : \ \Delta^{\mathcal{J}} = \Delta^{\mathcal{I}}, \cdot^{\mathcal{J}} = \cdot^{\mathcal{I}}$ *except that* $R^{\mathcal{J}} = R^{\mathcal{I}} \setminus \mathcal{R}$ *for some* $\mathcal{R} \subseteq R^{\mathcal{I}} \cap (C^{\mathcal{I}} \times \Delta^{\mathcal{I}})\}$.

4 Modular terminologies

If $\mathscr{R} \subseteq \mathfrak{Roles}$, $\mathscr{R} \neq \emptyset$, then we define

$$\mathcal{T}^{\mathscr{R}} = \{C \sqsubseteq D \in \mathcal{T} \ : \ \mathit{roles}(C \sqsubseteq D) \cap \mathscr{R} \neq \emptyset\}$$

Hence, $\mathcal{T}^{\mathscr{R}}$ contains all non-boolean axioms of the terminology $\mathcal{T}$ whose roles appear in $\mathscr{R}$. For $\mathscr{R} = \emptyset$, we define

$$\mathcal{T}^{\emptyset} = \{C \sqsubseteq D \in \mathcal{T} \ : \ \mathit{roles}(C \sqsubseteq D) = \emptyset\},$$

denoting the set of all boolean axioms of a knowledge base.

For example, if

$$\mathcal{T} = \left\{ \begin{array}{c} \mathsf{Passenger} \sqsubseteq \exists\mathsf{passport}.\top, \ \mathsf{EUcitizen} \equiv \forall\mathsf{passport}.\mathsf{EU}, \\ \mathsf{Foreigner} \equiv \forall\mathsf{passport}.\neg\mathsf{EU}, \ \mathsf{Foreigner} \sqsubseteq \exists\mathsf{refund}.\mathsf{Tax}, \\ \mathsf{DoubleCitizen} \equiv \mathsf{Foreigner} \sqcap \mathsf{EUcitizen} \end{array} \right\}$$

then we have

$$\mathcal{T}^{\{\mathsf{refund}\}} = \{\mathsf{Foreigner} \sqsubseteq \exists\mathsf{refund}.\mathsf{Tax}\}$$

and

$$\mathcal{T}^{\emptyset} = \{\mathsf{DoubleCitizen} \equiv \mathsf{Foreigner} \sqcap \mathsf{EUcitizen}\}$$

For parsimony's sake, we write $\mathcal{T}^{R}$ instead of $\mathcal{T}^{\{R\}}$.

We suppose from now on that $\mathcal{T}$ is *partitioned*, in the sense that $\{\mathcal{T}^{\emptyset}\} \cup \{\mathcal{T}^{R_i} \ : \ R_i \in \mathfrak{Roles}\}$ is a partition[2] of $\mathcal{T}$. We thus exclude $\mathcal{T}^{R_i}$ containing more than one role name, which means that complex concepts with nested roles are not allowed. We here rely on the following principle of modularity [11]:

[2]Remembering, $\{\mathcal{T}^{\emptyset}\} \cup \{\mathcal{T}^{R_i} \ : \ R_i \in \mathfrak{Roles}\}$ partitions $\mathcal{T}$ if and only if $\mathcal{T} = \mathcal{T}^{\emptyset} \cup \bigcup_{R_i \in \mathfrak{Roles}} \mathcal{T}^{R_i}$, and $\mathcal{T}^{\emptyset} \cap \mathcal{T}^{R_i} = \emptyset$, and $\mathcal{T}^{R_i} \cap \mathcal{T}^{R_j} = \emptyset$ if $i \neq j$. Note that $\mathcal{T}^{\emptyset}$ and $\mathcal{T}^{R_i}$ might be empty.

Definition 8 *A terminology $\mathcal{T}$ is* modular *if and only if for every $C \sqsubseteq D$,*

$$\mathcal{T} \models C \sqsubseteq D \text{ implies } \mathcal{T}^{roles(C \sqsubseteq D)} \cup \mathcal{T}^{\emptyset} \models C \sqsubseteq D.$$

Modularity means that when investigating whether $C \sqsubseteq D$ is a consequence of $\mathcal{T}$, the only axioms in $\mathcal{T}$ that are relevant are those whose role names occur in $C \sqsubseteq D$ and the boolean axioms in $\mathcal{T}^{\emptyset}$.

Modularity does not generally hold. Clearly if the TBox is not partitioned, then modularity fails. To witness, consider

$$\mathcal{T} = \{C \equiv \forall R_1.\forall R_2.C', \forall R_1.\forall R_2.C' \equiv D\}$$

Then $\mathcal{T} \models C \equiv D$, but $\mathcal{T}^{\emptyset} \not\models C \equiv D$.

Nevertheless even under the hypothesis that $\{\mathcal{T}^{\emptyset}\} \cup \{\mathcal{T}^{R_i} : R_i \in \mathfrak{Roles}\}$ partitions $\mathcal{T}$, modularity may fail to hold. For example, let

$$\mathcal{T} = \{C \sqsubseteq \forall R.\bot, C \sqsubseteq \exists R.\top\}$$

Then $\mathcal{T}^{\emptyset} = \emptyset$, and $\mathcal{T}^{R} = \mathcal{T}$. Now $\mathcal{T} \models C \sqsubseteq \bot$, but clearly $\mathcal{T}^{\emptyset} \not\models C \sqsubseteq \bot$.

How can we know whether a given TBox $\mathcal{T}$ is modular? The following criterion is simpler:

Definition 9 *A terminology $\mathcal{T}$ is* boolean-modular *if and only if for every boolean axiom $C \sqsubseteq D$,*

$$\mathcal{T} \models C \sqsubseteq D \text{ implies } \mathcal{T}^{\emptyset} \models C \sqsubseteq D.$$

This property is enough to guarantee modularity:

Theorem 1 ([11]) *Let $\mathcal{T}$ be a partitioned terminology. If $\mathcal{T}$ is boolean-modular, then $\mathcal{T}$ is modular.*

Modular TBoxes have several advantages. For example, consistency of a modular TBox can be checked by just checking consistency of $\mathcal{T}^{\emptyset}$: If $\mathcal{T}$ is modular, then $\mathcal{T} \models C \equiv \neg C$ if and only if $\mathcal{T}^{\emptyset} \models C \equiv \neg C$. Deduction of an axiom with nested roles $R_1; \ldots; R_n$ does not need to take into account the axioms with value restrictions for roles other than $R_1; \ldots; R_n$.

In [11] is given a sound and complete method for deciding whether a TBox is modular.

Similar modularity notions have also been proposed in the literature [2]. Modularity is related to uniform interpolation for TBoxes [7]. Let *concepts*($\mathcal{T}$) denote the concept names occurring in a TBox $\mathcal{T}$. Given $\mathcal{T}$ and a signature $\mathcal{S} \subseteq concepts(\mathcal{T}) \cup roles(\mathcal{T})$, a TBox $\mathcal{T}^{\mathcal{S}}$ over $(concepts(\mathcal{T}) \cup roles(\mathcal{T})) \setminus \mathcal{S}$ is a *uniform interpolant* of $\mathcal{T}$ outside $\mathcal{S}$ if and only if:

- $\mathcal{T} \models \mathcal{T}^{\mathcal{S}}$;
- $\mathcal{T}^{\mathcal{S}} \models C \sqsubseteq D$ for every $C \sqsubseteq D$ that has no occurrences of symbols from $\mathcal{S}$.

It is not difficult to see that a partition $\{\mathcal{T}^{\emptyset}\} \cup \{\mathcal{T}^{R_i} : R_i \in \mathfrak{Roles}\}$ is modular if and only if every $\mathcal{T}^{R_i}$ is a uniform interpolant of $\mathcal{T}$ outside $roles(\mathcal{T}) \setminus \{R_i\}$. In [19] there are complexity results for computing uniform interpolants in $\mathcal{ALC}$.

In [7] a notion of conservative extension is defined that is similar to modularity. There, $\mathcal{T}_1 \cup \mathcal{T}_2$ is a *conservative extension* of $\mathcal{T}_1$ if and only if for all concepts C, D built from $concepts(\mathcal{T}_1) \cup roles(\mathcal{T}_1)$, $\mathcal{T}_1 \cup \mathcal{T}_2 \models C \sqsubseteq D$ implies $\mathcal{T}_1 \models C \sqsubseteq D$.

Given Theorem 1, we can show that checking for modularity can be reduced to checking for conservative extensions of $\mathcal{T}^{\emptyset}$. Indeed, supposing that the signature of $\mathcal{T}^{\emptyset}$ is the set of all concept names, we have that $\mathcal{T}$ is modular if and only if for every role R_i, $\mathcal{T}^{R_i} \cup \mathcal{T}^{\emptyset}$ is a conservative extension of $\mathcal{T}^{\emptyset}$.

We now define a quantifier-based decomposition of TBoxes:

Definition 10 *Let $\mathcal{T}$ be a TBox satisfying Hypothesis (H). Then $\mathcal{T}_{\forall}^{\mathscr{R}} = \{C \sqsubseteq \forall R.D : C \sqsubseteq \forall R.D \in \mathcal{T}^{\mathscr{R}}\}$, and $\mathcal{T}_{\exists}^{\mathscr{R}} = \{C \sqsubseteq \exists R.D : C \sqsubseteq \exists R.D \in \mathcal{T}^{\mathscr{R}}\}$.*

Hence $\mathcal{T}_{\forall}^{\mathscr{R}}$ contains all axioms with value restrictions among $\mathscr{R}$ in the TBox $\mathcal{T}$, and $\mathcal{T}_{\exists}^{\mathscr{R}}$ all axioms with existential restrictions among $\mathscr{R}$ in $\mathcal{T}$.

With that we have decomposed a TBox $\mathcal{T}$ into modules $\mathcal{T}^{\emptyset}$, $\mathcal{T}_{\forall}^{\mathscr{R}}$ and $\mathcal{T}_{\exists}^{\mathscr{R}}$. We henceforth suppose that every axiom in $\mathcal{T}$ has the form of the axioms in one of these three modules.

5 Contracting terminologies

Having established the semantics of TBox contraction, we can turn to its syntactical counterpart.

Let $\mathcal{T} = \langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle$ be a terminology and $C \sqsubseteq D$ an axiom. Then by $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle^{-}_{C \sqsubseteq D}$ we denote the TBox resulting from the contraction of $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle$ by $C \sqsubseteq D$.

Contracting a TBox by a boolean axiom $C \sqsubseteq D$ amounts to using any existing contraction operator for classical logic. Let $\ominus$ be such an operator that is correct w.r.t. its semantic counterpart of Section 3. We define contraction of a terminology by a boolean axiom as follows:

Definition 11 $\langle \mathcal{T}^\emptyset, \mathcal{T}_\forall^\mathscr{R}, \mathcal{T}_\exists^\mathscr{R} \rangle^-_{C \sqsubseteq D} = \langle \mathcal{T}^{\emptyset^-}, \mathcal{T}_\forall^\mathscr{R}, \mathcal{T}_\exists^{\mathscr{R}^-} \rangle$, *where* $\mathcal{T}^{\emptyset^-} = \mathcal{T}^\emptyset \ominus C \sqsubseteq D$ *and* $\mathcal{T}_\exists^{\mathscr{R}^-} = \{C_i \sqcap (\neg C \sqcup D) \sqsubseteq \exists R.D_i \ : \ C_i \sqsubseteq \exists R.D_i \in \mathcal{T}_\exists^\mathscr{R}\} \cup \mathcal{T}_\exists^{\mathscr{R} \setminus \{R\}}$.

The reason we change $\mathcal{T}_\exists^\mathscr{R}$ is that we also need to guarantee that $C \sqsubseteq D$ does not follow from $\mathcal{T}_\forall^\mathscr{R}$ and $\mathcal{T}_\exists^\mathscr{R}$.[3]

We now consider the case of contracting a TBox by a simple axiom with existential restriction $C \sqsubseteq \exists R.D$. For every element of $\mathcal{T}_\exists^\mathscr{R}$, we ensure that $\exists R.D$ still subsumes those concepts $C' \sqcap \neg C$. The following operator does the job.

Definition 12 $\langle \mathcal{T}^\emptyset, \mathcal{T}_\forall^\mathscr{R}, \mathcal{T}_\exists^\mathscr{R} \rangle^-_{C \sqsubseteq \exists R.D} = \langle \mathcal{T}^\emptyset, \mathcal{T}_\forall^\mathscr{R}, \mathcal{T}_\exists^{\mathscr{R}^-} \rangle$, *where* $\mathcal{T}_\exists^{\mathscr{R}^-} = \{C_i \sqcap \neg C \sqsubseteq \exists R.D_i \ : \ C_i \sqsubseteq \exists R.D_i \in \mathcal{T}_\exists^\mathscr{R}\} \cup \mathcal{T}_\exists^{\mathscr{R} \setminus \{R\}}$.

For instance, contracting the axiom $\mathsf{Foreigner} \sqcap \mathsf{DoubleCitizen} \sqsubseteq \exists \mathsf{refund.Tax}$ from our TBox example gives us $\mathsf{Foreigner} \sqcap \neg \mathsf{DoubleCitizen} \sqsubseteq \exists \mathsf{refund.Tax}$ in the result.

Finally, to contract a terminology by $C \sqsubseteq \forall R.D$, for every axiom in $\mathcal{T}_\forall^\mathscr{R}$ we must ensure that all concepts $C' \sqcap \neg C$ are still specializations of $\forall R.D$. This is enough to guarantee that the axiom $C \sqsubseteq \forall R.D$ has been contracted. The operator below formalizes this:

Definition 13 $\langle \mathcal{T}^\emptyset, \mathcal{T}_\forall^\mathscr{R}, \mathcal{T}_\exists^\mathscr{R} \rangle^-_{C \sqsubseteq \forall R.D} = \langle \mathcal{T}^\emptyset, \mathcal{T}_\forall^{\mathscr{R}^-}, \mathcal{T}_\exists^\mathscr{R} \rangle$, *where* $\mathcal{T}_\forall^{\mathscr{R}^-} = \{C_i \sqcap \neg C \sqsubseteq \forall R.D_i \ : \ C_i \sqsubseteq \forall R.D_i \in \mathcal{T}_\forall^\mathscr{R}\} \cup \mathcal{T}_\forall^{\mathscr{R} \setminus \{R\}}$.

For instance, contracting $\mathsf{DoubleCitizen} \sqsubseteq \forall \mathsf{passport.EU}$ from our TBox weakens $\mathsf{EUcitizen} \sqsubseteq \forall \mathsf{passport.EU}$ to $(\mathsf{EUcitizen} \sqcap \mathsf{DoubleCitizen}) \sqsubseteq \forall \mathsf{passport.EU}$ in the contracted TBox.

6 Characterization results

In this section we present the main results that follow from our framework. These require the TBox under consideration to be modular.

Here we establish that our operators are correct w.r.t. the semantics. Our first theorem establishes that the semantical contraction of the models of $\langle \mathcal{T}^\emptyset, \mathcal{T}_\forall^\mathscr{R}, \mathcal{T}_\exists^\mathscr{R} \rangle$ by $C \sqsubseteq D$ produces models of $\langle \mathcal{T}^\emptyset, \mathcal{T}_\forall^\mathscr{R}, \mathcal{T}_\exists^\mathscr{R} \rangle^-_{C \sqsubseteq D}$.

[3] An example of such an inference is $\mathcal{T}_\forall^\mathscr{R} = \{C \sqsubseteq \forall R.D\}$ and $\mathcal{T}_\exists^\mathscr{R} = \{C \sqsubseteq \exists R.\neg D\}$: we have $\mathcal{T}_\forall^\mathscr{R} \cup \mathcal{T}_\exists^\mathscr{R} \models C \sqsubseteq \bot$. Thus $C \sqsubseteq \bot$ is what has been called an implicit boolean inclusion axiom of $\mathcal{T}$ [11].

Theorem 2 *Let $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ be a model of $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle$, and let $C \sqsubseteq D$ be a simple axiom. For all models $\mathcal{J}$, if $\mathcal{J} \in \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle^{-}_{C \sqsubseteq D}$, then $\mathcal{J}$ satisfies $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle^{-}_{C \sqsubseteq D}$.*

It remains now to prove that the models of $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle^{-}_{C \sqsubseteq D}$ result from the semantical contraction of models of $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle$ by $C \sqsubseteq D$. This does not hold in general, as shown by the following example: suppose there is only one atomic concept A and one atomic role R, and consider the TBox $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle$ such that $\mathcal{T}^{\emptyset} = \emptyset$, $\mathcal{T}_{\forall}^{\mathscr{R}} = \{A \sqsubseteq \forall R.\bot, A \sqsubseteq \forall R.A\}$, and $\mathcal{T}_{\exists}^{\mathscr{R}} = \{\top \sqsubseteq \exists R.\top\}$. The only models of this TBox are $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, where $\Delta^{\mathcal{I}} = (\neg A)^{\mathcal{I}}$ and $R^{\mathcal{I}} = (\neg A)^{\mathcal{I}} \times (\neg A)^{\mathcal{I}}$, and by definition, $\mathcal{I}^{-}_{A \sqsubseteq \exists R.\top} = \{\mathcal{I}\}$. On the other hand, syntactically, $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle^{-}_{A \sqsubseteq \exists R.\top} = \langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \{\neg A \sqsubseteq \exists R.\top\} \rangle$. The contracted TBox has *two* models: $\mathcal{I}$ and $\mathcal{J} = \langle \Delta^{\mathcal{J}}, \cdot^{\mathcal{J}} \rangle$, where $R^{\mathcal{J}} = (\neg A)^{\mathcal{J}} \times (\neg A)^{\mathcal{J}}$. While $\neg A$ is valid in the contraction of the models of $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle$, it is not valid in the models of $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle^{-}_{A \sqsubseteq \exists R.\top}$.

Fortunately, we can establish a result for those TBoxes that are modular. The proof requires three lemmas. The first one says that for a modular TBox we can restrict our attention to its 'big' models.

Lemma 1 *Let $\mathcal{T} = \langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle$ be modular, and let $C \sqsubseteq D$ be a boolean axiom. Then $\mathcal{T} \models C \sqsubseteq D$ if and only if κ-model$(\mathcal{T}^{\emptyset}) \models C \sqsubseteq D$.*

Note that the lemma does not hold for non-modular TBoxes.

The second lemma says that modularity is preserved under contraction.

Lemma 2 *Let $\mathcal{T}$ be modular, and $C \sqsubseteq D$ a simple axiom. Then $\mathcal{T}^{-}_{C \sqsubseteq D}$ is modular.*

The third lemma establishes the required link between the contraction operators and contraction of 'big' models.

Lemma 3 *Let $\mathcal{T} = \langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle$ be modular, and $C \sqsubseteq D$ be a simple axiom. If $\mathcal{J} = \langle \kappa\text{-model}(\mathcal{T}^{\emptyset}), \cdot^{\mathcal{J}} \rangle$ is a model of $\mathcal{T}^{-}_{C \sqsubseteq D}$, then there is a model $\mathcal{I}$ of $\mathcal{T}$ such that $\mathcal{J} \in \mathcal{I}^{-}_{C \sqsubseteq D}$.*

Putting the three above lemmas together we get:

Theorem 3 *Let $\mathcal{T} = \langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle$ be modular, let $C \sqsubseteq D$ be a simple axiom, and $\langle \mathcal{T}^{\emptyset^-}, \mathcal{T}_{\forall}^{\mathscr{R}^-}, \mathcal{T}_{\exists}^{\mathscr{R}^-} \rangle$ be $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle^{-}_{C \sqsubseteq D}$. If $\mathcal{T}^{\emptyset^-}, \mathcal{T}_{\forall}^{\mathscr{R}^-}, \mathcal{T}_{\exists}^{\mathscr{R}^-} \models C' \sqsubseteq D'$, then for every model $\mathcal{I}$ of $\mathcal{T}$ and every $\mathcal{J} \in \mathcal{I}^{-}_{C \sqsubseteq D}$, $\mathcal{J} \models C' \sqsubseteq D'$.*

Our two theorems together establish correctness of the operators:

Corollary 1 *Let $\mathcal{T} = \langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle$ be modular, $C \sqsubseteq D$ be a simple axiom, and $\langle \mathcal{T}^{\emptyset^-}, \mathcal{T}_{\forall}^{\mathscr{R}^-}, \mathcal{T}_{\exists}^{\mathscr{R}^-} \rangle$ be $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle^-_{C \sqsubseteq D}$. Then $\mathcal{T}^{\emptyset^-}, \mathcal{T}_{\forall}^{\mathscr{R}^-}, \mathcal{T}_{\exists}^{\mathscr{R}^-} \models C' \sqsubseteq D'$ if and only if for every model $\mathcal{I}$ of $\mathcal{T}$ and every $\mathcal{J} \in \mathcal{I}^-_{C \sqsubseteq D}$, $\mathcal{J} \models C' \sqsubseteq D'$.*

We give a necessary condition for success of contraction:

Theorem 4 *Let $C \sqsubseteq D$ be a simple non-boolean axiom such that $\mathcal{T}^{\emptyset} \not\models C \sqsubseteq D$. Let $\langle \mathcal{T}^{\emptyset^-}, \mathcal{T}_{\forall}^{\mathscr{R}^-}, \mathcal{T}_{\exists}^{\mathscr{R}^-} \rangle$ be $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle^-_{C \sqsubseteq D}$. If $\langle \mathcal{T}^{\emptyset}, \mathcal{T}_{\forall}^{\mathscr{R}}, \mathcal{T}_{\exists}^{\mathscr{R}} \rangle$ is modular, then $\mathcal{T}^{\emptyset^-}, \mathcal{T}_{\forall}^{\mathscr{R}^-}, \mathcal{T}_{\exists}^{\mathscr{R}^-} \not\models C \sqsubseteq D$.*

7 Concluding remarks

In this work we have presented a method for changing a TBox given an axiom we want to contract. We have supposed that the axioms in the TBox and those to be contracted are simple.

We have defined a semantics for terminology contraction and also presented its syntactical counterpart through contraction operators. Soundness and completeness of such operators with respect to the semantics have been established (Corollary 1).

We have also shown that modularity is a sufficient condition for a contraction to be successful (Theorem 4).

We are aware that our contraction operators are rather weak, in the sense that e.g. when we contract by $C \sqsubseteq \forall R.D$ then we forget about all the information regarding value restrictions for role R. We think that this is the price to pay for a domain-independent terminology-change operation. If we want to go beyond and define refined change operations then it seems that we have to move toward more syntax-dependent operations, as studied in the belief revision literature [8, 15].

What is the status of the AGM-postulates for contraction in our framework? First, contraction of boolean axioms satisfies all the postulates, as soon as we assume the underlying classical contraction operation $\ominus$ satisfies all of them.

In the general case, however, our constructions do not satisfy the central postulate of preservation $\mathcal{T}^-_{C \sqsubseteq D} = \mathcal{T}$ if $\mathcal{T} \not\models C \sqsubseteq D$. Indeed, suppose we have a language with only one atomic concept A, and a model $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ such that $R^{\mathcal{I}} = (\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}) \setminus (A^{\mathcal{I}} \times A^{\mathcal{I}})$. Then $\mathcal{I} \models A \sqsubseteq \forall R.\neg A$ and $\mathcal{I} \not\models \top \sqsubseteq \forall R.\neg A$. Now the contraction $\mathcal{I}^-_{\top \sqsubseteq \forall R.\neg A}$ yields the model $\mathcal{J}$ such that $R^{\mathcal{J}} = (\Delta^{\mathcal{J}} \times \Delta^{\mathcal{J}})$.

Then $\mathcal{J} \not\models A \sqsubseteq \forall R.\neg A$, i.e., the axiom $A \sqsubseteq \forall R.\neg A$ is not preserved. Our contraction operation thus behaves rather like an update operation.

Now let us focus on the other postulates. Since our operator has a behavior which is close to the update postulate, we focus on the following basic erasure postulates introduced in [13]. Let $Cn(\mathcal{T})$ be the set of all logical consequences of a TBox $\mathcal{T}$.

KM1 $Cn(\mathcal{T}^-_{C \sqsubseteq D}) \subseteq Cn(\mathcal{T})$

Postulate **KM1** does not always hold because it is possible to make an axiom $C \sqsubseteq \forall R.\bot$ valid in the resulting TBox by removing elements of $R^{\mathcal{I}}$ (cf. Definition 7).

KM2 $C \sqsubseteq D \notin Cn(\mathcal{T}^-_{C \sqsubseteq D})$

Under the condition that $\mathcal{T}$ is modular, Postulate **KM2** is satisfied (cf. Theorem 4).

KM3 If $Cn(\mathcal{T}_1) = Cn(\mathcal{T}_2)$ and $Cn(\{C \sqsubseteq D\}) = Cn(C' \sqsubseteq D'\})$, then $Cn(\mathcal{T}^-_{1 C' \sqsubseteq D'}) = Cn(\mathcal{T}^-_{2 C \sqsubseteq D})$.

Theorem 5 *If $\mathcal{T}_1$ and $\mathcal{T}_2$ are modular and the propositional contraction operator $\ominus$ satisfies Postulate* **KM3***, then Postulate* **KM3** *is satisfied for all simple axioms $C \sqsubseteq D$ and $C' \sqsubseteq D'$.*

Here we have presented the case for contraction, but our definitions can be extended to expansion and revision of TBoxes, too.

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