Formal Foundations of Ontologies and Reasoning

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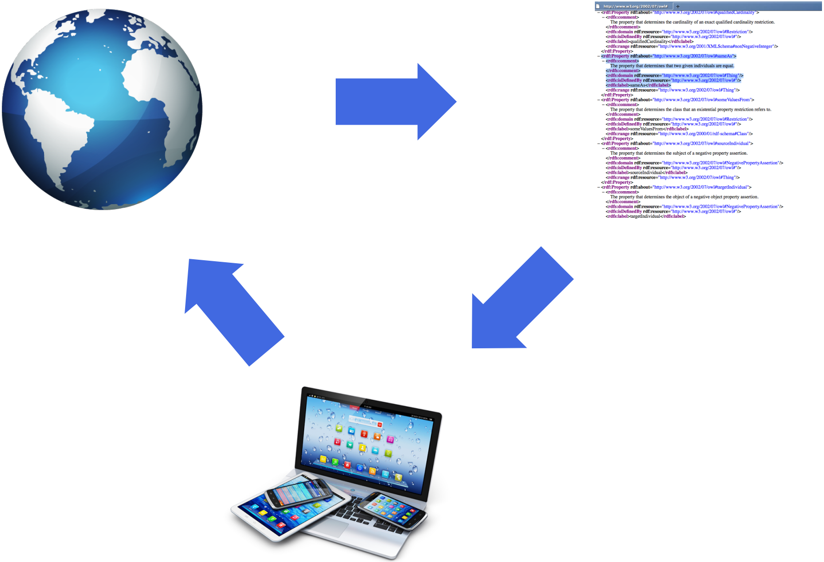
http://www.ijv.ovh

Why are we here?



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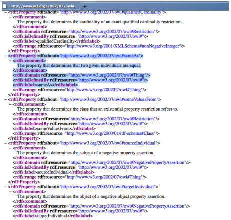

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http://www.w3.org/2002/07/owl#
– <rdf:Property rdf:about="http://www.w3.org/2002/07/owl#qualifiedCardinality">
  – <rdfs:comment>
      The property that determines the cardinality of an exact qualified cardinality restriction.
    </rdfs:comment>
    <rdfs:domain rdf:resource="http://www.w3.org/2002/07/owl#Restriction"/>
    <rdfs:isDefinedBy rdf:resource="http://www.w3.org/2002/07/owl#"/>
    <rdfs:label>qualifiedCardinality</rdfs:label>
    <rdfs:range rdf:resource="http://www.w3.org/2001/XMLSchema#nonNegativeInteger"/>
  </rdf:Property>
– <rdf:Property rdf:about="http://www.w3.org/2002/07/owl#sameAs">
  – <rdfs:comment>
      The property that determines that two given individuals are equal.
    </rdfs:comment>
    <rdfs:domain rdf:resource="http://www.w3.org/2002/07/owl#Thing"/>
    <rdfs:isDefinedBy rdf:resource="http://www.w3.org/2002/07/owl#"/>
    <rdfs:label>sameAs</rdfs:label>
    <rdfs:range rdf:resource="http://www.w3.org/2002/07/owl#Thing"/>
  </rdf:Property>
– <rdf:Property rdf:about="http://www.w3.org/2002/07/owl#someValuesFrom">
  – <rdfs:comment>
      The property that determines the class that an existential property restriction refers to.
    </rdfs:comment>
    <rdfs:domain rdf:resource="http://www.w3.org/2002/07/owl#Restriction"/>
    <rdfs:isDefinedBy rdf:resource="http://www.w3.org/2002/07/owl#"/>
    <rdfs:label>someValuesFrom</rdfs:label>
    <rdfs:range rdf:resource="http://www.w3.org/2000/01/rdf-schema#Class"/>
  </rdf:Property>
– <rdf:Property rdf:about="http://www.w3.org/2002/07/owl#sourceIndividual">
  – <rdfs:comment>
      The property that determines the subject of a negative property assertion.
    </rdfs:comment>
    <rdfs:domain rdf:resource="http://www.w3.org/2002/07/owl#NegativePropertyAssertion"/>
    <rdfs:isDefinedBy rdf:resource="http://www.w3.org/2002/07/owl#"/>
    <rdfs:label>sourceIndividual</rdfs:label>
    <rdfs:range rdf:resource="http://www.w3.org/2002/07/owl#Thing"/>
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– <rdf:Property rdf:about="http://www.w3.org/2002/07/owl#targetIndividual">
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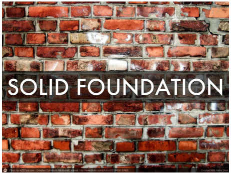

Overview of the course

Main parts

1. Introduction to ontologies and description logics
2. The description logic $\mathcal{ALC}$
3. Introduction to modelling and reasoning with $\mathcal{ALC}$
4. Reasoning with ontologies
5. More and less expressive DLs
6. Formal ontologies in OWL and Protégé

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There will be

- Examples
- Exercises
- A lot of interaction (I hope)

Overview of the course

Bibliography

- F. Baader, D. Calvanese, D. McGuinness, D. Nardi, and P. Patel-Schneider (eds.): The Description Logic Handbook: Theory, Implementation and Applications. Cambridge University Press, 2nd edition, 2007.
- F. Baader, I. Horrocks, C. Lutz, and U. Sattler. An Introduction to Description Logic. Cambridge University Press, 2017.
- M. Krötzsch, F. Simančík, and I. Horrocks. Description Logic Primer. `http://arxiv.org/pdf/1201.4089v3.pdf`
- The Protégé Ontology Editor. `http://protege.stanford.edu`
- The Description Logic workshop series. `http://dl.kr.org`

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Course website

- `https://tinyurl.com/Graz2019DL`

Outline of Part 1

Formal Ontologies

Introduction to DLs

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Ontologies

Explicit specification of a shared conceptualisation

Example (The student ontology)

- Employed students are students and employees
- Students are not taxpayers (do not pay taxes)
- Employed students are taxpayers (pay taxes)
- Employed students who are parents are not taxpayers (do not pay taxes)
- To work for is to be employed by
- John is an employed student, John works for IBM

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- John is an employed student, John and IBM are in works for

classes relations individuals
specialisation and instantiation

Main ingredients in formal ontologies

A common vocabulary and a shared understanding

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Classes or concepts

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Instances of classes and relations

- Name objects of the domain and denote representatives of a concept
- E.g.: John, John is an employed student, John works for IBM

Why Description Logics?

Expressivity

- Concepts ✓
- Relations ✓
- Instances ✓

DLs have all one needs to formalise ontologies!

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- Amenability to implementation ✓
- Decidability ✓
- Good trade-off between expressivity and complexity ✓

Most DL-based systems satisfy all of these!

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Available tools

protégé

FaCT++

Pellet

HermiT

CEL

...

Outline of Part 1

First of all, what are DLs?

Decidability

- Some logics can be made decidable by sacrificing expressive power
- DLs are less expressive than full first-order logic
- DLs are decidable, but what complexity is “OK”?

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Technically

- DLs are a family of fragments of first-order logic
- Only two variable names
- For the *cognoscenti*: correspond to guarded fragments of FOL
- But much, much simpler than FOL. . .

Elements of the language (domain dependent)

Atomic concept names

- $\mathsf{C} =_{\text{def}} \{A_1, \ldots, A_n\}$ (Special concepts: $\top$, $\bot$)
- Intuition: basic classes of a domain of interest
- Student, Employee, Parent

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Individual names

- I $=_{\text{def}} \{a_1, \ldots, a_l\}$
- Intuition: names of objects in the domain
- john, mary, ibm

Elements of the language (domain independent)

Boolean constructors

- Concept negation: $\neg$ (class complement)
- Concept conjunction: $\sqcap$ (class intersection)
- Concept disjunction: $\sqcup$ (class union)

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Further constructors: cardinality constraints, inverse roles, ... (if needed)

Building concepts

Definition (Complex concepts)

- $\top$ and $\bot$ are concepts
- Every concept name $A \in \mathsf{C}$ is a concept
- If C and D are concepts and $r \in \mathsf{R}$, then

$\neg C$	(complement of C)	$\exists r.C$	(existential restriction)
$C \sqcap D$	(intersection of C and D)	$\forall r.C$	(value restriction)
$C \sqcup D$	(union of C and D)		

  are all concepts
- Nothing else is a concept (at least for now)

Exercise

Which ones are concepts and which aren't?

- $\top \sqcap \bot \sqcup \top$
- $C \sqcup \forall r. \sqcap \neg D$
- $C \sqcup \neg\neg\exists D$
- $\exists r.\top$
- $\exists r.\forall s.C \sqcap D$
- $\forall r.C \sqcap \neg D$
- $\forall r.(C \sqcap \neg D)$
- $\forall\exists r.C$

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Building concepts

Full negation

- Negation of arbitrary concepts
- Intuition: the complement of a concept
- E.g.: $\neg\neg\mathsf{Student}$ $\quad \neg(\mathsf{Student} \sqcap \mathsf{Parent})$

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Atomic negation

- Some DLs only allow negation of concept names
- Good complexity results
- E.g.: $\neg\mathsf{Student}$ $\quad\neg\mathsf{Parent}$

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Concept conjunction

- Intuition: the intersection of two concepts
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So far we have seen the Boolean fragment of our concept language

- At least as expressive as classical propositional logic

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Existential restriction

- Intuition: there is some link with a concept
- E.g.: $\exists \mathsf{pays}.\mathsf{Tax}$

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So far we have got $\mathcal{ALC}$ (Attributive Language with Complement)

- Prototypical concept description language (there are others)

Language

Different flavours

- $\mathcal{ALC}$: $C ::= \top \mid \bot \mid \mathsf{C} \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \forall r.C \mid \exists r.C$
- $\mathcal{ALCQ}$: $C ::= \ldots \mid \geq nr.C \mid \leq nr.C$
- $\mathcal{EL}$, DL-Lite, $\mathcal{SHIQ}$, $\mathcal{SHOQ}$, $\mathcal{SROIQ}$ (basis of OWL 2), ...

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Example

$\neg(\mathsf{Student} \sqcap \mathsf{Parent})$	$\mathsf{Student} \sqcap \neg\exists \mathsf{pays}.\mathsf{Tax}$
$\exists \mathsf{empBy}.\mathsf{Company}$	$\mathsf{EmpStud} \sqcap \exists \mathsf{pays}.\mathsf{Tax}$
$\mathsf{Employee} \sqcup \mathsf{Student} \sqcap \exists \mathsf{worksFor}.\mathsf{Parent}$	$\forall \mathsf{worksFor}.\mathsf{Company}$

Language

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$\mathsf{Employee} \sqcup \mathsf{Student} \sqcap \exists \mathsf{worksFor}.\mathsf{Parent}$

$\mathsf{Student} \sqcap \neg\exists \mathsf{pays}.\mathsf{Tax}$

$\mathsf{EmpStud} \sqcap \exists \mathsf{pays}.\mathsf{Tax}$

$\forall \mathsf{worksFor}.\mathsf{Company}$

With $\mathcal{L}_{\mathcal{ALC}}$ we denote the concept language of $\mathcal{ALC}$

Semantics

Definition (Interpretation)

Tuple $\mathcal{I} =_{\text{def}} \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, where

- $\Delta^{\mathcal{I}}$ is a domain (set of objects)
- $\cdot^{\mathcal{I}}$ is an interpretation function such that

$$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \qquad r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \qquad a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$$

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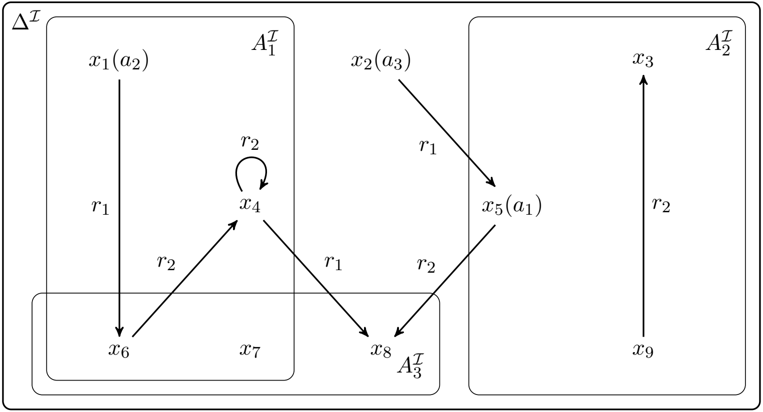
Example

Let $\mathsf{C} = \{A_1, A_2, A_3\}$, $\mathsf{R} = \{r_1, r_2\}$, $\mathsf{I} = \{a_1, a_2, a_3\}$. Let $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ where:

- $\Delta^{\mathcal{I}} = \{x_i \mid 1 \leq i \leq 9\}$, $a_1^{\mathcal{I}} = x_5$, $a_2^{\mathcal{I}} = x_1$, $a_3^{\mathcal{I}} = x_2$
- $A_1^{\mathcal{I}} = \{x_1, x_4, x_6, x_7\}$, $A_2^{\mathcal{I}} = \{x_3, x_5, x_9\}$, $A_3^{\mathcal{I}} = \{x_6, x_7, x_8\}$
- $r_1^{\mathcal{I}} = \{(x_1, x_6), (x_4, x_8), (x_2, x_5)\}$, $r_2^{\mathcal{I}} = \{(x_4, x_4), (x_6, x_4), (x_5, x_8), (x_9, x_3)\}$

Semantics

$\mathcal{I}:$



Semantics

Extending DL interpretations

$$\top^{\mathcal{I}} =_{\text{def}} \Delta^{\mathcal{I}} \quad \bot^{\mathcal{I}} =_{\text{def}} \emptyset \quad (\neg C)^{\mathcal{I}} =_{\text{def}} \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$$

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$$(\exists r.C)^{\mathcal{I}} =_{\text{def}} \{x \in \Delta^{\mathcal{I}} \mid r^{\mathcal{I}}(x) \cap C^{\mathcal{I}} \neq \emptyset\}$$

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Semantics

Extending DL interpretations

$$\top^{\mathcal{I}} =_{\text{def}} \Delta^{\mathcal{I}} \quad \bot^{\mathcal{I}} =_{\text{def}} \emptyset \quad (\neg C)^{\mathcal{I}} =_{\text{def}} \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$$

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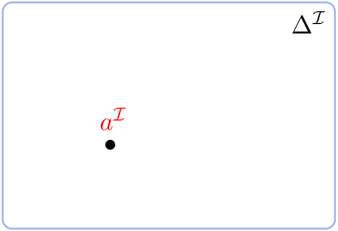
Definition (Concept Satisfiability)

A concept C is satisfiable if there is $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ s.t. $C^{\mathcal{I}} \neq \emptyset$

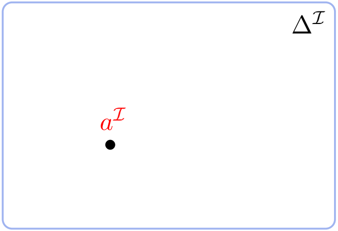
Semantics



Individual names

- At most one element of $\Delta^{\mathcal{I}}$

Semantics



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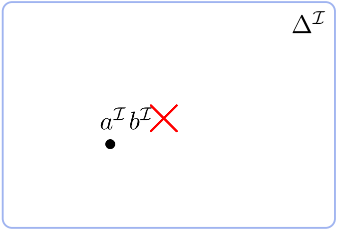


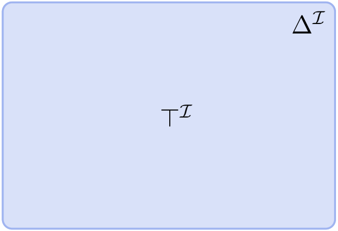
Unique Name Assumption

- At most one name per object

Semantics

The 'top' concept

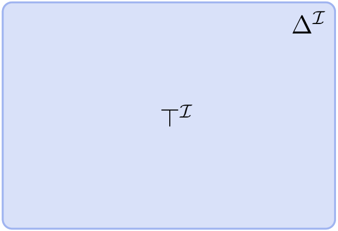
- Everything is in $\top^{\mathcal{I}}$
- Also called Thing



Semantics

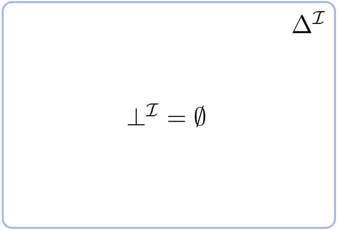
The 'top' concept

- Everything is in $\top^{\mathcal{I}}$
- Also called Thing



The 'bottom' concept

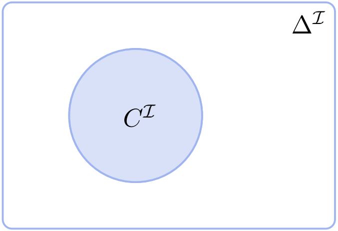
- $\bot^{\mathcal{I}}$ is in everything
- Also called Nothing



Semantics

Arbitrary concept

- A class in the domain
- $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$



Semantics

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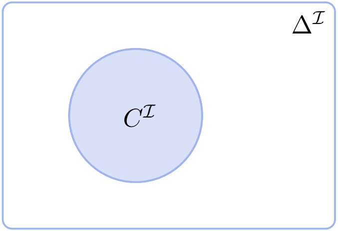
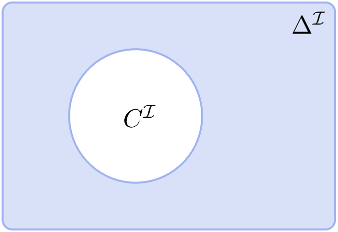
- A class in the domain
- $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$



Concept negation

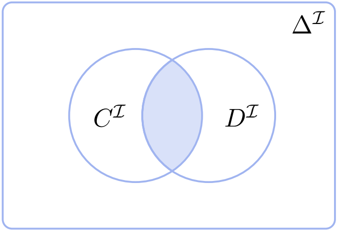
- The complement of a concept
- $(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$



Semantics

Concept conjunction

- The intersection of two classes
- $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$



Semantics

Concept conjunction

- The intersection of two classes
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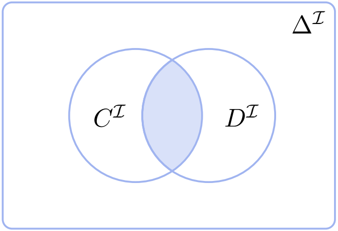
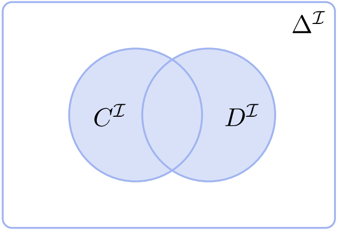


Concept disjunction

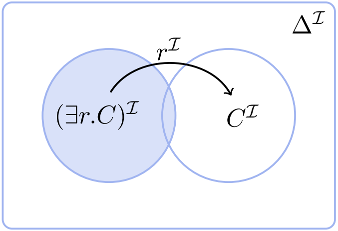
- The union of two classes
- $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$



Semantics

Existential restriction

- At least one value of a class
- $(\exists r.C)^{\mathcal{I}} = \{x \mid r^{\mathcal{I}}(x) \cap C^{\mathcal{I}} \neq \emptyset\}$



Semantics

Existential restriction

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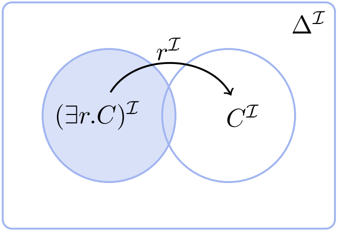
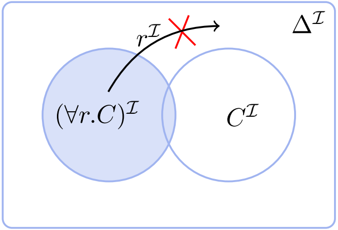


Value restriction

- All values of a class
- $(\forall r.C)^{\mathcal{I}} = \{x \mid r^{\mathcal{I}}(x) \subseteq C^{\mathcal{I}}\}$



Semantics

An interpretation is a complete description of the world

$\mathcal{I}$: $\Delta^{\mathcal{I}}$

x_0 x_1 x_2 x_3

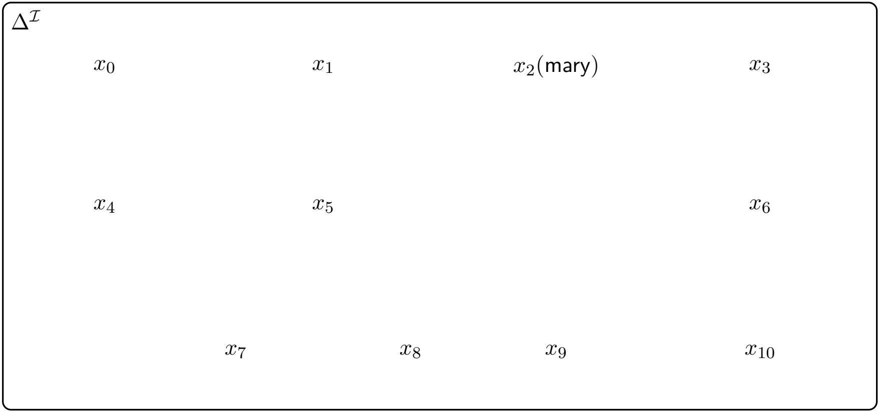
x_4 x_5 x_6

x_7 x_8 x_9 x_{10}

Semantics

An interpretation is a complete description of the world

$\mathcal{I}$:



Semantics

An interpretation is a complete description of the world

$\mathcal{I}$: $\Delta^{\mathcal{I}}$

x_0 x_1 x_2(mary) x_3

x_4 x_5(john) x_6

x_7 x_8 x_9 x_{10}

Semantics

An interpretation is a complete description of the world

$\mathcal{I}$: $\Delta^{\mathcal{I}}$

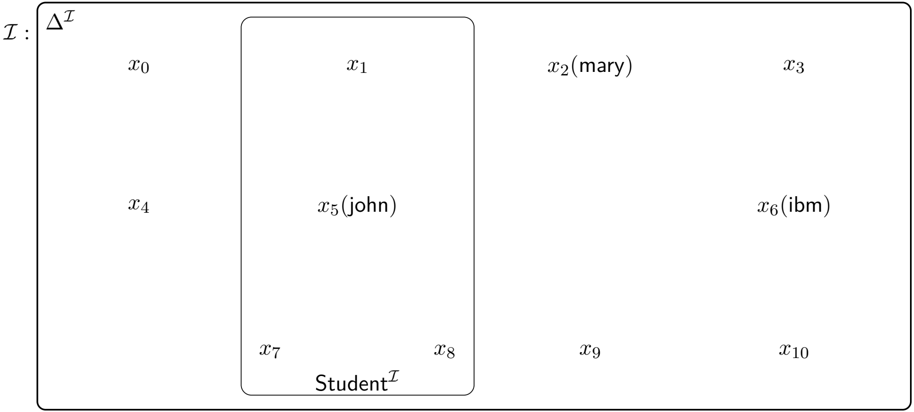
x_0 x_1 x_2(mary) x_3

x_4 x_5(john) x_6(ibm)

x_7 x_8 x_9 x_{10}

Semantics

An interpretation is a complete description of the world



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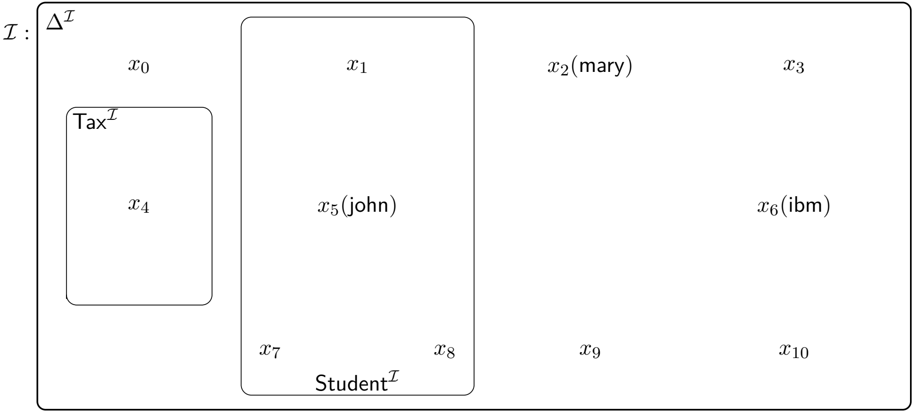

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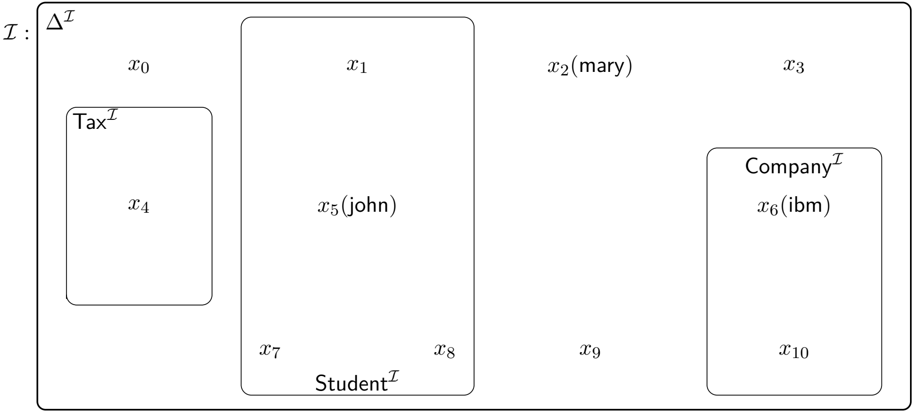

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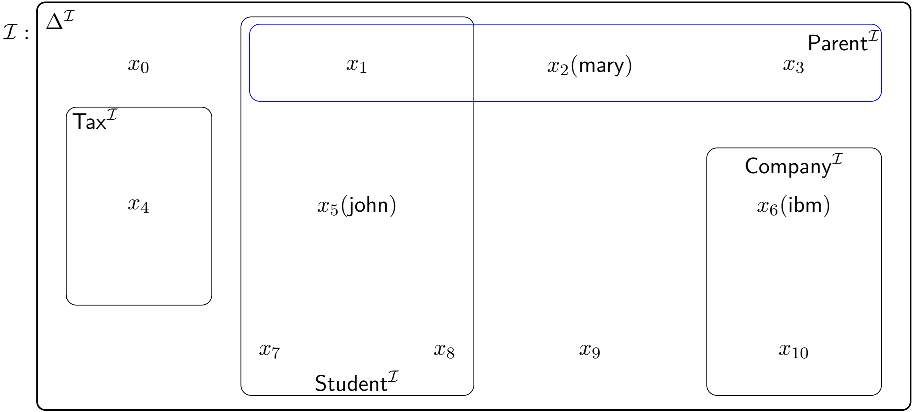

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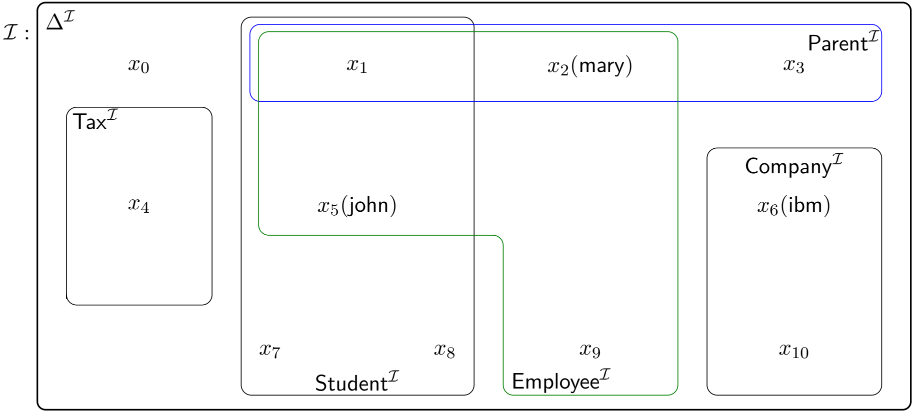

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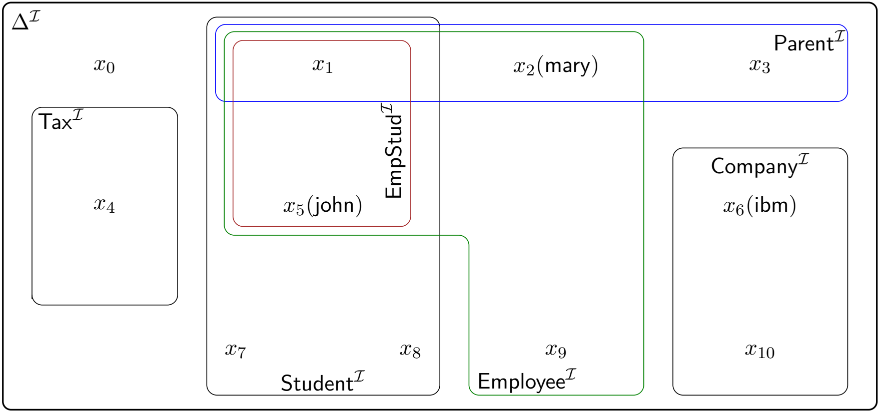

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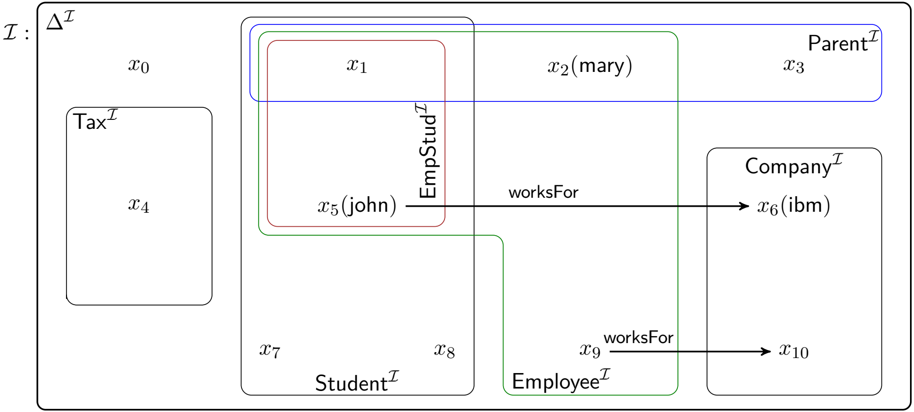

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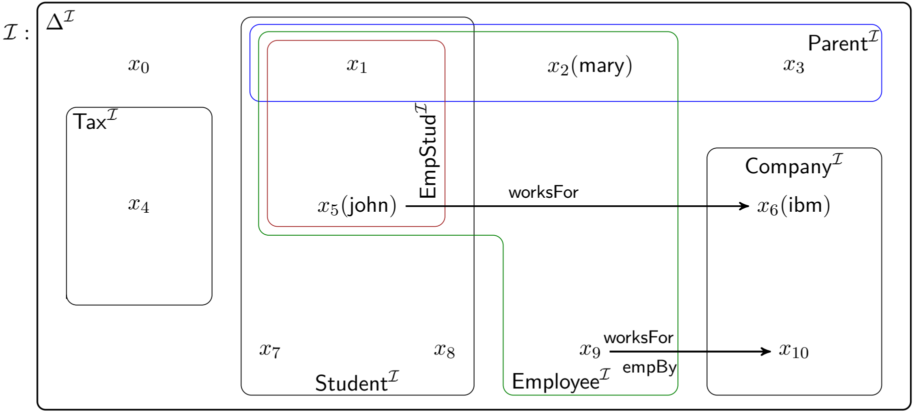

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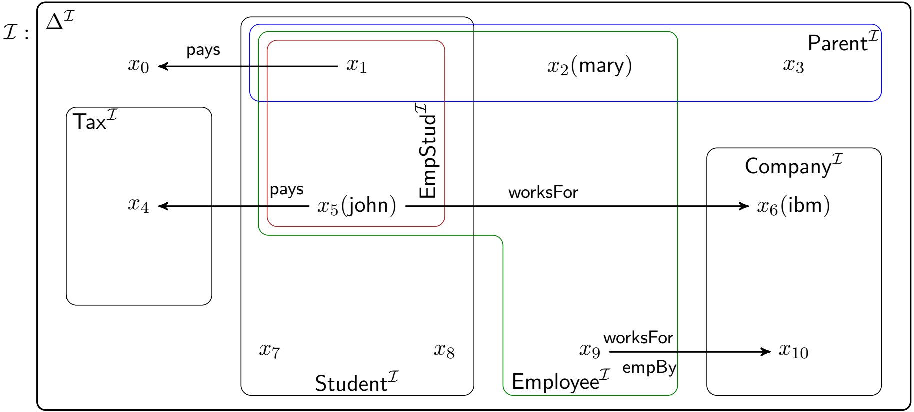

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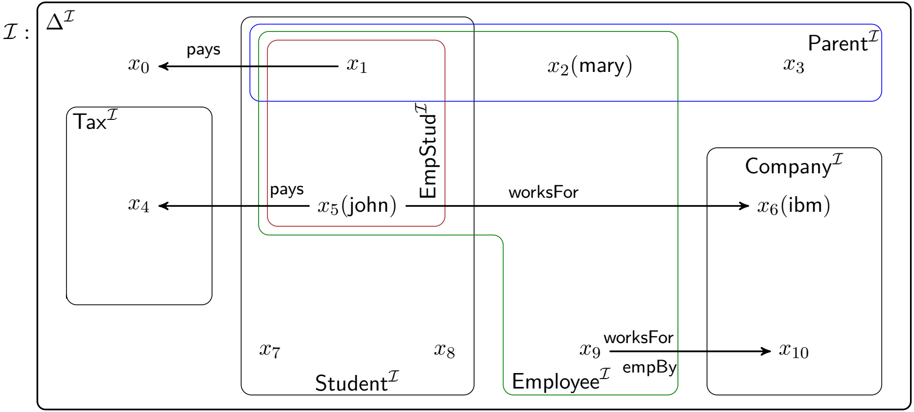
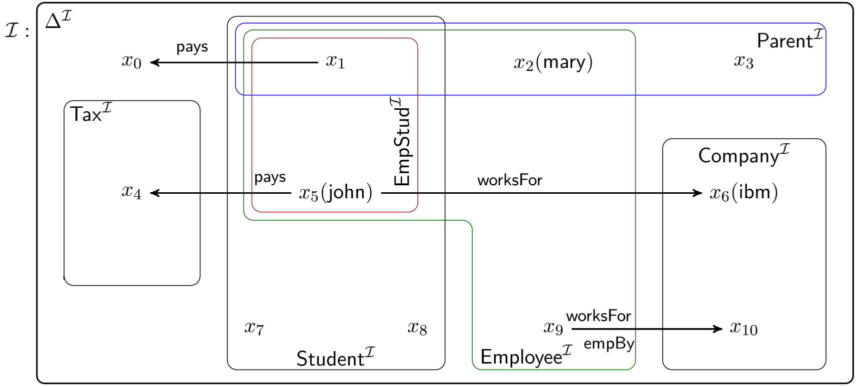


$$((\mathsf{EmpStud} \sqcup \mathsf{Parent}) \sqcap \exists \mathsf{pays}.\top)^{\mathcal{I}} = \{x_1, x_5\}$$

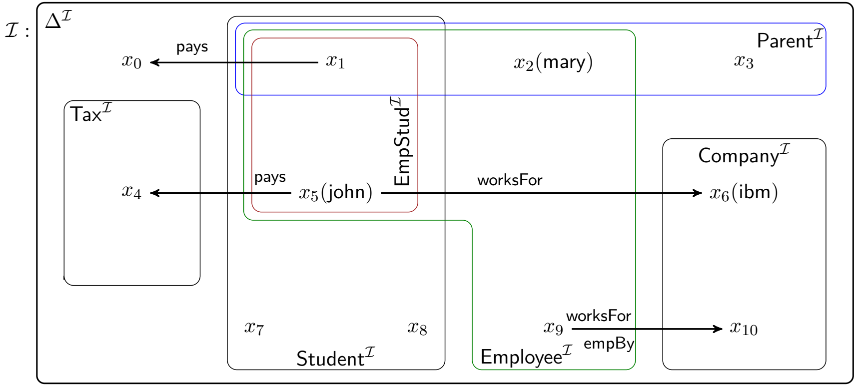
Exercise

Let $\mathsf{C} = \{\mathsf{Company}, \mathsf{Employee}, \mathsf{EmpStud}, \mathsf{Parent}, \mathsf{Student}, \mathsf{Tax}\}$, $\mathsf{R} = \{\mathsf{empBy}, \mathsf{pays}, \mathsf{worksFor}\}$ $\mathsf{I} = \{\mathsf{ibm}, \mathsf{john}, \mathsf{mary}\}$



Exercise

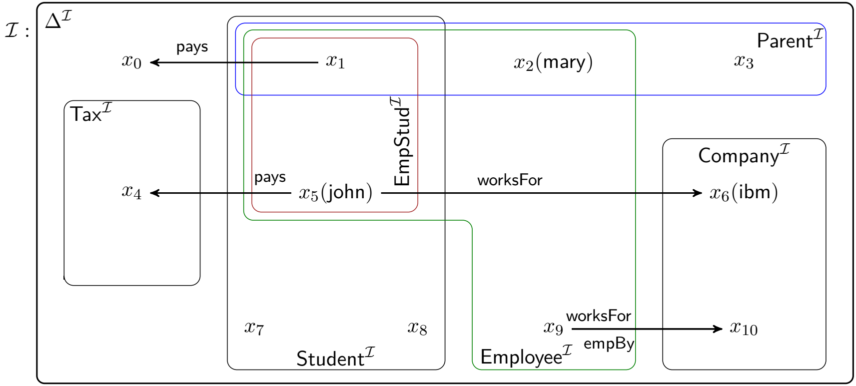
Let C = {Company, Employee, EmpStud, Parent, Student, Tax}, R = {empBy, pays, worksFor} I = {ibm, john, mary}



- $(\neg\text{Employee})^{\mathcal{I}} = ?$
- $(\exists\text{pays}.\top)^{\mathcal{I}} = ?$
- $(\neg\text{Parent} \sqcap \text{Employee})^{\mathcal{I}} = ?$
- $(\neg\text{EmpStud} \sqcap \forall\text{empBy}.\text{Company})^{\mathcal{I}} = ?$
- $(\exists\text{worksFor}.\exists\text{empBy}.\text{Parent})^{\mathcal{I}} = ?$
- $(\text{Student} \sqcap \forall\text{pays}.\bot)^{\mathcal{I}} = ?$

Exercise

Let $\mathsf{C} = \{\mathsf{Company}, \mathsf{Employee}, \mathsf{EmpStud}, \mathsf{Parent}, \mathsf{Student}, \mathsf{Tax}\}$, $\mathsf{R} = \{\mathsf{empBy}, \mathsf{pays}, \mathsf{worksFor}\}$ $\mathsf{I} = \{\mathsf{ibm}, \mathsf{john}, \mathsf{mary}\}$



- $(\neg\mathsf{Employee})^{\mathcal{I}} = \{x_0, x_3, x_4, x_6, x_7, x_8, x_{10}\}$
- $(\exists\mathsf{pays}.\top)^{\mathcal{I}} = ?$
- $(\neg\mathsf{Parent} \sqcap \mathsf{Employee})^{\mathcal{I}} = ?$
- $(\neg\mathsf{EmpStud} \sqcap \forall\mathsf{empBy}.\mathsf{Company})^{\mathcal{I}} = ?$
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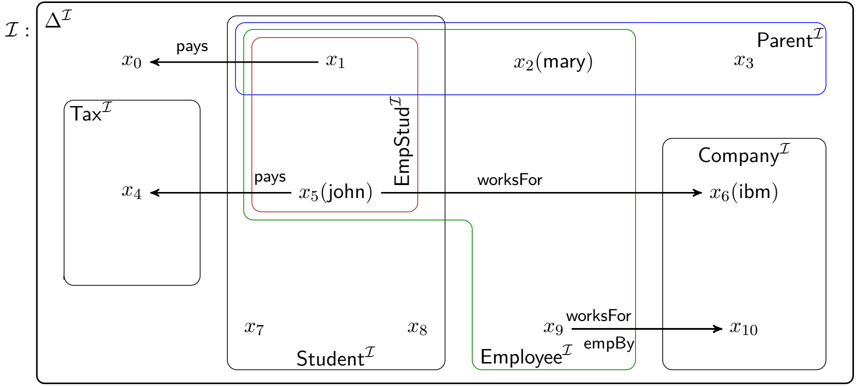

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- $(\exists\mathsf{pays}.\top)^{\mathcal{I}} = \{x_1, x_5\}$
- $(\neg\mathsf{Parent} \sqcap \mathsf{Employee})^{\mathcal{I}} =$?
- $(\neg\mathsf{EmpStud} \sqcap \forall\mathsf{empBy}.\mathsf{Company})^{\mathcal{I}} =$?
- $(\exists\mathsf{worksFor}.\exists\mathsf{empBy}.\mathsf{Parent})^{\mathcal{I}} =$?
- $(\mathsf{Student} \sqcap \forall\mathsf{pays}.\bot)^{\mathcal{I}} =$?

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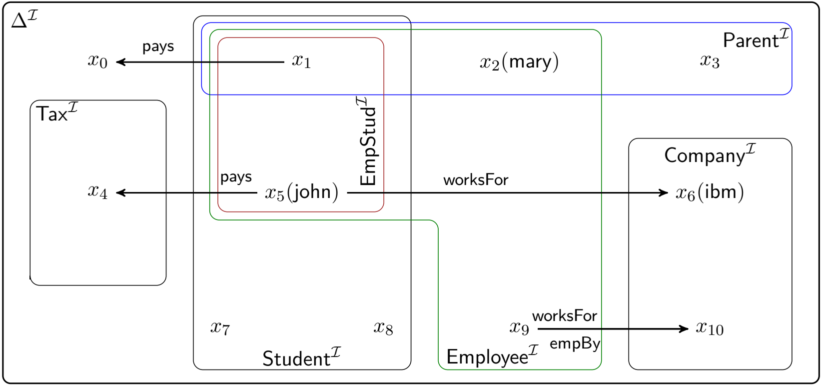
Let $\mathsf{C} = \{\mathsf{Company}, \mathsf{Employee}, \mathsf{EmpStud}, \mathsf{Parent}, \mathsf{Student}, \mathsf{Tax}\}$, $\mathsf{R} = \{\mathsf{empBy}, \mathsf{pays}, \mathsf{worksFor}\}$ $\mathsf{I} = \{\mathsf{ibm}, \mathsf{john}, \mathsf{mary}\}$

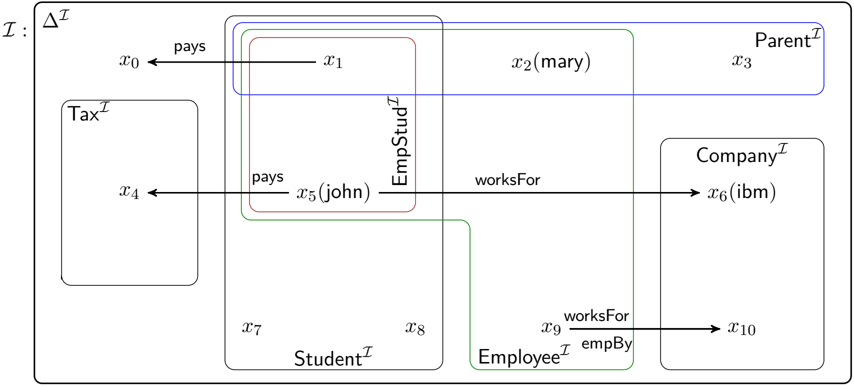
$\mathcal{I}$:



- $(\neg\mathsf{Employee})^{\mathcal{I}} = \{x_0, x_3, x_4, x_6, x_7, x_8, x_{10}\}$
- $(\exists\mathsf{pays}.\top)^{\mathcal{I}} = \{x_1, x_5\}$
- $(\neg\mathsf{Parent} \sqcap \mathsf{Employee})^{\mathcal{I}} = \{x_5, x_9\}$
- $(\neg\mathsf{EmpStud} \sqcap \forall\mathsf{empBy}.\mathsf{Company})^{\mathcal{I}} = ?$
- $(\exists\mathsf{worksFor}.\exists\mathsf{empBy}.\mathsf{Parent})^{\mathcal{I}} = ?$
- $(\mathsf{Student} \sqcap \forall\mathsf{pays}.\bot)^{\mathcal{I}} = ?$

Exercise

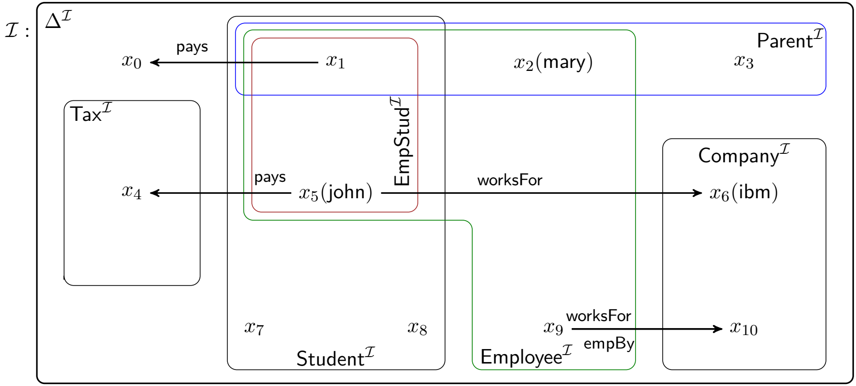
Let C = {Company, Employee, EmpStud, Parent, Student, Tax}, R = {empBy, pays, worksFor} I = {ibm, john, mary}



- $(\neg \text{Employee})^{\mathcal{I}} = \{x_0, x_3, x_4, x_6, x_7, x_8, x_{10}\}$
- $(\exists \text{pays}.\top)^{\mathcal{I}} = \{x_1, x_5\}$
- $(\neg \text{Parent} \sqcap \text{Employee})^{\mathcal{I}} = \{x_5, x_9\}$
- $(\neg \text{EmpStud} \sqcap \forall \text{empBy}.\text{Company})^{\mathcal{I}} = \{x_9\}$
- $(\exists \text{worksFor}.\exists \text{empBy}.\text{Parent})^{\mathcal{I}} = ?$
- $(\text{Student} \sqcap \forall \text{pays}.\bot)^{\mathcal{I}} = ?$

Exercise

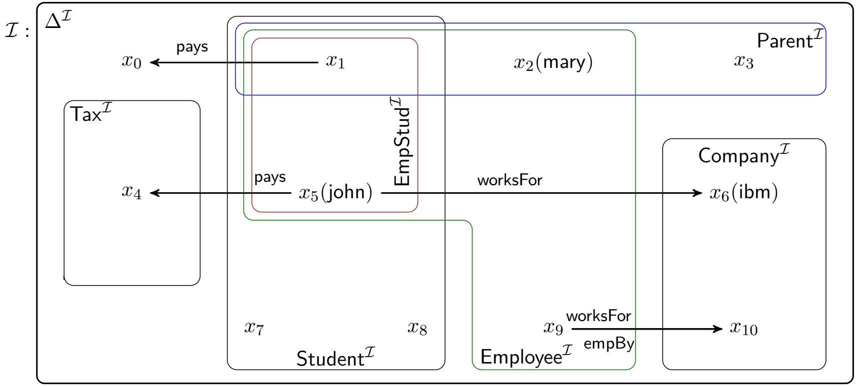
Let C = {Company, Employee, EmpStud, Parent, Student, Tax}, R = {empBy, pays, worksFor} I = {ibm, john, mary}



- $(\neg\text{Employee})^{\mathcal{I}} = \{x_0, x_3, x_4, x_6, x_7, x_8, x_{10}\}$
- $(\exists\text{pays}.\top)^{\mathcal{I}} = \{x_1, x_5\}$
- $(\neg\text{Parent} \sqcap \text{Employee})^{\mathcal{I}} = \{x_5, x_9\}$
- $(\neg\text{EmpStud} \sqcap \forall\text{empBy}.\text{Company})^{\mathcal{I}} = \{x_9\}$
- $(\exists\text{worksFor}.\exists\text{empBy}.\text{Parent})^{\mathcal{I}} = \emptyset$
- $(\text{Student} \sqcap \forall\text{pays}.\bot)^{\mathcal{I}} = ?$

Exercise

Let C = {Company, Employee, EmpStud, Parent, Student, Tax}, R = {empBy, pays, worksFor} I = {ibm, john, mary}



- $(\neg\text{Employee})^{\mathcal{I}} = \{x_0, x_3, x_4, x_6, x_7, x_8, x_{10}\}$
- $(\exists\text{pays}.\top)^{\mathcal{I}} = \{x_1, x_5\}$
- $(\neg\text{Parent} \sqcap \text{Employee})^{\mathcal{I}} = \{x_5, x_9\}$
- $(\neg\text{EmpStud} \sqcap \forall\text{empBy}.\text{Company})^{\mathcal{I}} = \{x_9\}$
- $(\exists\text{worksFor}.\exists\text{empBy}.\text{Parent})^{\mathcal{I}} = \emptyset$
- $(\text{Student} \sqcap \forall\text{pays}.\bot)^{\mathcal{I}} = \{x_7, x_8\}$

Exercise

Let $\mathsf{C} = \{\mathsf{Company}, \mathsf{Employee}, \mathsf{EmpStud}, \mathsf{Parent}, \mathsf{Student}, \mathsf{Tax}\}$, $\mathsf{R} = \{\mathsf{empBy}, \mathsf{pays}, \mathsf{worksFor}\}$ $\mathsf{I} = \{\mathsf{ibm}, \mathsf{john}, \mathsf{mary}\}$

Find an interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ such that:

- $(\mathsf{Student} \sqcap \mathsf{Employee})^{\mathcal{I}} = \emptyset$, $\mathsf{Parent}^{\mathcal{I}} \subseteq (\mathsf{Student} \sqcup \mathsf{Employee})^{\mathcal{I}}$, $(\neg\mathsf{EmpStud})^{\mathcal{I}} = \Delta^{\mathcal{I}}$
- $\mathsf{Student}^{\mathcal{I}} \subseteq (\forall \mathsf{pays}.\bot)^{\mathcal{I}}$, $(\exists \mathsf{worksFor}.\top)^{\mathcal{I}} \subseteq (\neg(\mathsf{Student} \sqcup \mathsf{Tax} \sqcup \mathsf{Company}))^{\mathcal{I}}$, $\mathsf{Employee}^{\mathcal{I}} \subseteq (\exists \mathsf{empBy}.\top)^{\mathcal{I}}$

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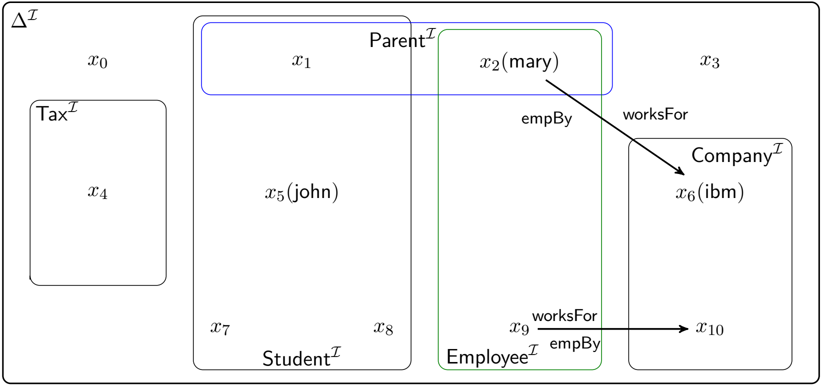
$\mathcal{I}$:



Some Properties

Lemma

For every interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, and for every $C, D \in \mathcal{L}_{\mathcal{ALC}}$

- $(\neg\neg C)^{\mathcal{I}} = C^{\mathcal{I}}$
- $(\neg(C \sqcap D))^{\mathcal{I}} = (\neg C \sqcup \neg D)^{\mathcal{I}}$
- $(\neg(C \sqcup D))^{\mathcal{I}} = (\neg C \sqcap \neg D)^{\mathcal{I}}$
- $(\neg\forall r.C)^{\mathcal{I}} = (\exists r.\neg C)^{\mathcal{I}}$
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$\mathcal{ALC}$ is the smallest propositionally closed DL

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Theorem

$\mathcal{ALC}$ has the finite model property: if C is satisfiable, then there is $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ such that $C^{\mathcal{I}} \neq \emptyset$ and $\Delta^{\mathcal{I}}$ is finite

Epilogue

Summary

- What we mean by ontology
- Formal ontologies and their main ingredients
- Basic description logics
- The concept language and its semantics
- How DLs relate to other formalisms

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What next?

- A fundamental notion in DLs
- Formalising ontologies with DLs